High School Math Badging System

Alignment Guidance for Illustrative Mathematics

Developed for XQ by Student Achievement Partners

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Introduction

About these Materials

This set of curriculum guidance documents is designed to inform the use of the Algebra 1, Geometry, Algebra 2 Illustrative Mathematics Curriculum in the implementation of the High School Badging System developed for XQ Institute by Student Achievement Partners.

This curriculum was selected because of its strong alignment to the vision of the Common Core Math Standards, as evidenced by EdReports' review, and because it is licensed under the Creative Commons Attribution 4.0 license, making it freely available at Kendall-Hunt Publishing Company.

About Our Process

These materials were developed by a team of educators using Illustrative Mathematics and/or the High School Badging System. Our team worked collaboratively to identify units and lessons aligned to the Badging System's Content and Practice Expectations by badge number.

Grounded in the Badging System's Framework Principles, our team also worked to leverage the structure and content of the curriculum to optimize opportunities for students to engage in the Badging System's Learning Principles.

Each document identifies the corresponding Illustrative Mathematics' course and unit materials and is comprised of three parts:

- Notes for improving overall alignment
- Unit Overview
- Guidance for Lesson Priority

What follows is a description of each component in order to provide educator teams with a way to make sense of and engage with these materials.

Notes for improving overall alignment

This section consists of broad recommendations that apply across the board. Readers will see two types of recommendations: (1) ways to better attend to the Badging Learning Principles and (2) ways to better attend to Content and Practice Expectations in cases where we saw limited opportunities built into the curricular materials.

Sample

M152: Reasoning and Proof through Congruence

Illustrative Mathematics–Geometry Unit 2-Congruence

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum and opportunities to use the embedded math language routines to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- Educators will need to bring in space to attend to Indicator 152.a.i. Consider using Geometry Unit 1 Lessons 10, 13-14 to offer students space to use these skills. Spaces where we see an opportunity for this are indicated by a square ■.
- Educators will need to bring in space to attend to Indicator 152.j.i and 152.j.ii. Consider using Geometry Unit 1 Lesson 5 to offer students space to use these skills. Spaces where we see an opportunity for this are indicated by a triangle ▲.

Unit Overview

This section consists of a table that illustrates when students have an opportunity to engage with each Content and Practice Expectation at the indicator level. It gives educators a birds-eye view of the story of the unit. It is important to note that lessons early in the unit may be at the introductory level and mastery is not expected. For CPEs with a short storyline, we encourage educators to utilize the embedded practice problems offered by the curriculum to give students additional opportunities as needed. Applicable practice problems will need to be identified at the local level.

Sample

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	MT
155.a.i	X	X	X	X	X							X
155.a.ii				X	X							
155.b.i								X				
155.b.ii								X		X		
155.c.i					X	X	X		X	X	X	X
155.c.ii									X	X		X

Guidance for Lesson Priority

This section consists of a table that lists each lesson and offers more details to inform lesson design. We recognize that local needs may differ and additional factors may need to be taken into account as educators make instructional and curricular decisions. Recommendations found in this table are to be considered as a starting point for a team's consideration.

Educators will find the following information provided:

- **Unit and lesson:** 5.7 refers to Unit 5 Lesson 7. On occasions when units from multiple courses are brought together to meet the expectations of a particular badge, the appropriate course will be communicated.
- **Priority:** A letter of D, M, or K is used to communicate our recommendation. Insights into these codes and our rationale is as follows:
    - K= Keep. We saw strong alignment to the CPEs and Learning Principles. Minimal, if any, modifications need to be done.
    - M= Modify. We saw an opportunity to make space for additional CPEs and/or stronger alignment to our Learning Principles with some modification. Recommendations are offered for your consideration.
    - D= Delete. We saw limited to no connection to the badge's CPEs.
- **Content and Practice Expectations (CPE)**: A list of the CPEs seen in the lesson. Information is provided at the indicator level.
- **Learning Principles (LP)**: A list of the Learning Principles available in the lesson.
- **Additional Lesson Information**: Gives description of the **context** and **math focus** of the lesson.

Sample

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.1	D Not needed to enter Lesson 5.2. Genie context extension not aligned with LPs.			**Context:** Bacteria growth, genie wishes **Math focus:** Linear and non-linear growth basics, review of grades 6,7,8
5.2	M Prioritize Activity 5.2.3 to give students an opportunity to see math as relevant (LP6), especially if students express an interest in use of social media. Doing so may mean skipping Activity 5.2.2.	104.b.i 104.b.ii 104.e.i 104.e.ii	LP3, LP6	**Context:** Social media **Math Focus:** Understanding the difference between exponential and linear
5.3	K	104.bi 104.b.ii 104.c.i 104.c.ii 104.e.ii 104.f.i	LP3	**Context:** Bacteria growth **Math Focus:** Multiple ways to represent exponential relationships

Glossary of Terms Used

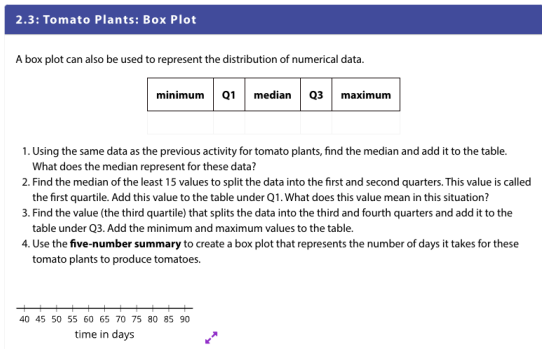
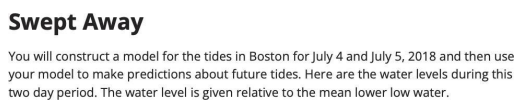
Terminology our team worked to define that you'll see surface across these documents are as follows:

- **Leverage structure of curriculum**

- IM is designed to have learning arcs which start with an invitation to the mathematics by way of a warm-up, followed by a deep study of concepts by way of two-three instructional activities, culminating in time to consolidate and synthesize learning by way of a lesson synthesis and cool-down. You can learn more about the curriculum's design principles at Kendall-Hunt's website.
- IM offers embedded instructional routines and deliberate instructional design to ensure all students have multiple means of engagement, representation, and action and expression in alignment with the principles of Universal Design for Learning. We urge educators to utilize these routines and structures in service of their students. This will not only ensure access to the content for all students, but will also offer ways to attend to the Badging System's Learning Principles. You can learn more about access for multilingual learners and students with disabilities at Kendall-Hunt's website.

- **Amplify**
  - Used to invite educators to consider centering the suggested activity to optimize connections to identified Content and Practice Expectations (CPE) and/or Learning Principles. In these cases, another instructional activity within the lesson will need to be de-emphasized or skipped. In making these recommendations, we worked to honor the coherence built into the curriculum.
    - In cases where an activity is amplified in order to better attend to the Learning Principles, we envision multiple possibilities: (1) giving students more time to truly engage and connect with the context and/or (2) using the mathematical focus and suggested instructional strategies, but paired with a local context students can relate to. An example of each is illustrated in the table below. For additional ideas on structuring experiences that attend to the Learning Principles, see the Annotated Examples provided in each badging framework.

Engaging students more deeply into the context
A1 Unit 1 Lesson 2 Activity 2: Tomato Plants: Histogram

<table>
<tr><td> 
</td><td>Our Recommendation:
Consider asking students to consider why anyone would want to know the number of days it takes for a tomato plant to start producing fruit as a launch to the activities in this lesson to attend LP6.</td></tr>
<tr><td colspan="2">Engaging students by selecting a local context</td></tr>
<tr><td colspan="2">A2 Unit 6 Modeling Task- Swept Away</td></tr>
<tr><td> 
</td><td>Our Recommendation:
Consider using local data to increase degree of relevance.</td></tr>
</table>

- In cases where an activity is amplified in order to better attend to the identified CPEs, we recognized a way to give entry into the mathematics by adding or modifying synthesizing questions offered within the curriculum teacher narrative.

Additional Resources for Illustrative Mathematics Curriculum

- Learn more about the IM Mathematical Modeling Prompts: https://im.kendallhunt.com/HS/teachers/mathematical_modeling_prompts.html
- Access additional curricular resources at IM Resource Hub for more ideas: https://hub.illustrativemathematics.org/s/912

M101 Linear Equations: Concepts and Skills

Illustrative Mathematics–Algebra 1 Unit 2-Linear Equations, Inequalities, and Systems

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- Educators will need to bring in space to attend to the second part of Indicator 101.d.iii.. Spaces where we see an opportunity for this are indicated by a triangle ▲.
- Badging sites may combine badges M101 and M102 into a single badge if your current curricula makes it difficult to disentangle the concepts and skills portion of linear functions and equations from the modeling portion.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense

of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
101.a: Reason about and solve one-variable linear equations and inequalities.	i. use variables to represent numbers and write equations when solving a real-world or mathematical problem.
	ii. demonstrate understanding that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set.
101.b: Solve real-life and mathematical problems using linear expressions, equations, and inequalities.	i. solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form while using tools strategically.
	ii. construct simple equations and inequalities to solve problems by reasoning about quantities.
101.c: Analyze and solve two-variable linear equations and pairs of simultaneous linear equations.	i. solve linear equations in two-variables.
	ii. give examples of linear equations in two-variables with one solution, an infinity of solutions, or no solutions.
	iii. analyze and solve pairs of simultaneous linear equations.
101.d: Represent and solve linear equations and inequalities graphically.	i. explain why the x-coordinates of the points where the graphs of the linear equations y=f(x) and y=g(x) intersect are the solutions to the equation f(x)=g(x).
	ii. find solutions to a system of linear equations in the form y=f(x) and y=g(x) approximately by using their graphs.
	iii. graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	L22	L24
101.a.i	X	X		X		X	X																
101.a.ii	X	X	X	X		X	X											X	X				
101.b.i				X	X				X	X		X							X	X		X	X
101.b.ii	X	X	X	X	X	X			X	X								X	X	X	X	X	X
101.c.i			X	X	X				X	X		X											
101.c.ii												X					X						
101.c.iii												X	X	X	X	X	X						
101.d.i												X	X	X	X	X	X						X
101.d.ii												X	X										X
101.d.iii																					X	X	X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.1	M Skip 1.3 Good lesson to set stage for use of mathematics to solve real-life and mathematical problems	101a.i 101.a.ii 101.b.ii	LP2, LP6	**Context:** Planning a pizza party **Math focus:** Using equations to represent situations
2.2	M Skip Activity 2.2 to give space for deeper discussion and engagement in Activity 2.3.	101a.i 101.a.ii 101.b.ii	LP2, LP6	**Context:** Platonic solids, blueberries, car prices **Math focus:** Using equations to represent situations
2.3	K	101.a.ii 101.b.ii 101.c.i	LP2, LP3	**Context:** Running, walking from home, electricity bills, monthly salary **Math focus:** Writing equations to model relationships
2.4	K	101a.i 101.a.ii 101.b.i 101.b.ii 101.c.i	LP1, LP3, LP6	**Context:** Earning money, calories and protein **Math focus:** Equations with one variable and two variables

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.5	M For Activity 5.3 - Have students choose between #1 or #2	101.b.i 101.b.ii 101.c.i	LP4, LP6, LP7	**Context:** Snacks at a grocery store, savings, pay check, draining water from a tank **Math focus:** Graphing equations
2.6	M For Activity 6.3 - Assign a few equations per table group	101.a.i 101.a.ii 101.b.ii	LP2, LP3, LP5	**Context:** Buying jeans with a discount, family and ages **Math focus:** Equivalent equations
2.7	K	101.a.i 101.a.ii	LP1, LP3	**Context:** Purely mathematical **Math focus:** Equivalent equations
2.8	D Better fit for M102- Modeling with Linear Functions and Equations			**Context:** Parade clean up, filling/emptying tanks **Math focus:** Rearrange equations to solve for a specific variable
2.9	M Skip Activity 9.1	101.b.i 101.b.ii 101.c.i	LP3, LP7	**Context:** Cargo shipping, city resurfacing roads and hiring additional staff **Math focus:** Rearrange equations to solve for a specific variable

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.10	M Prioritize Activity 10.2 to highlight the relationship between an equation and its graph. This means skip Activity 10.3. This lesson has connections to Badge M102.	101.b.i 101.b.ii 101.c.i	LP2	**Context:** Games and rides at a carnival **Math focus:** Connecting equations to graphs
2.11	D Better fit for M102- Modeling with Linear Functions and Equations			**Context:** Gallons in a tank, almonds and figs **Math focus:** Connecting equations to graphs
2.12	M Please note this lesson is also included in Badge M102 for focus on aspects of mathematical modeling. Consider having students engage in this through a modeling lens, then return to it with a lens on the CPEs listed.	101.b.i 101.c.i 101.c.iii 101.d.i 101.d.ii	LP1, LP3, LP7	**Context:** Trail mix, dining, fall, family tickets to a magic show, posters of favorite bands **Math focus:** Writing and graphing system of equations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.13	K Make graphing technology available (LP7) to give students opportunity to use graphs (101.d.ii) with Activity 13.1 and 13.2	101.c.iii 101.d.i 101.d.ii	LP1, LP2, LP7	**Context:** Purely mathematical **Math focus:** Solving system of equations by substitution
2.14	K	101.c.iii 101.d.i	LP1, LP3, LP7	**Context:** Purely mathematical **Math focus:** Solving system of equations by elimination
2.15	K	101.c.iii 101.d.i	LP2, LP3	**Context:** Teacher buying class supplies, reimbursement **Math focus:** Solving system of equations by elimination
2.16	M Prioritize Activity 16.3, space to communicate with peers and critique the reasoning of others (MP3). This may mean skipping 16.2.	101.c.iii 101.d.i	LP2, LP7	**Context:** Purely mathematical **Math focus:** Solving system of equations by elimination

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.17	M Consider using Activity 17.3's *Are you ready for more?* prompt as an additional opportunity for students to engage with Indicator 101.c.ii. Skip Activity 17.2, it is a better fit for Badge M102.	101.c.ii 101.c.iii 101.d.i	LP2, LP3, LP7	**Context:** Pool passes and gym membership **Math focus:** Number of solutions for systems of equations
2.18	M Please note this lesson is also included in Badge M102 for focus on aspects of mathematical modeling. Consider having students engage in this through a modeling lens, then return to it with a lens on the CPEs listed.	101.a.ii 101.b.ii	LP3	**Context:** Budgeting for a senior ball **Math focus:** Writing inequalities
2.19	M Please note this lesson is also included in Badge M102 for focus on	101.a.ii 101.b.i 101.b.ii	LP3, LP7	**Context:** Class field trip to an orchard, working to make money for college tuition **Math focus:** Solutions to one-variable

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	aspects of mathematical modeling. Consider having students engage in this through a modeling lens, then return to it with a lens on the CPEs listed.			inequalities
2.20	K	101.b.i 101.b.ii	LP1, LP2, LP3	**Context:** Buying food for rehearsal for drama club, mowing a lawn **Math focus:** Writing and solving inequalities with one-variable
2.21	K	101.b.ii 101.d.iii	LP2	**Context:** Purely mathematical **Math focus:** Graphing linear inequalities in two-variables
2.22	K	101.b.i 101.b.ii 101.d.iii	LP2, LP3, LP7	**Context:** Homeowner making plans to landscape a yard, vendors making profit at a market, raising money for a charity concert **Math focus:** Graphing linear inequalities in two-variables

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.24	M Complete Activity 24.4 if time allows.	101.b.i 101.b.ii 101.d.iii	LP5, LP6, LP7	**Context:** Buying fabric to make a quilt, opening bank accounts, types of concert tickets, packages for advertising services **Math focus:** Writing and graphing systems of linear inequalities in two-variables

M102: Modeling with Linear Functions and Equations

Illustrative Mathematics–Algebra 1 Unit 2: Linear Equations, Inequalities, and Systems **Illustrative Mathematics–Algebra 1 Unit 3: Two-variable statistics**

Notes for improving overall alignment:

- Consider attending to M101 and M102 concurrently. One way to engage students in recognizing the learning they are doing is to give students time to reflect on their learning in connection with the Content and Practice Expectations of this badge as assignments related to this badge surface, i.e., as opportunities to engage in mathematical modeling arise.
- Educators will need to bring in space to attend to Content and Practice Expectation 102.i. Consider using this template to offer students space to reflect and document their thinking at key points in time. Spaces where we see an opportunity for this are indicated by a triangle ▲.
- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of Math Language Routine 5: Co-Craft Questions may be useful in building these opportunities into some of the existing modeling tasks within the unit, particularly some of the more complex tasks toward the end of the unit.
- Badging sites may combine badges M101 and M102 into a single badge if your current curricula makes it difficult to

disentangle the concepts and skills portion of linear functions and equations from the modeling portion.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental

challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
102.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
102.b: Interpret linear functions and equations that arise in applications in terms of the context.	i. select two different data points that illustrate something interesting or important about the context you are modeling and describe what these points tell you about the topic.
	ii. use a model to compute output values and then interpret the values in the context of the problem.
	iii. describe the domain of the linear function, including any noticings about values that aren't part of the domain given the context.
	iv. describe the contextual meaning of the rate of change for a linear model.

Content and Practice Expectations	Indicators Choose an artifact where you…
102.c: Analyze linear functions using different representations.	i. create a linear graph to show a particular context and describe important values that help to understand something about the problem you are exploring.
	ii. explain how the parameters of a linear model relate to its graph.
102.d: Build linear functions that model relationships between two quantities.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate a linear relationship.
	ii. build a function that models the linear relationship between two quantities in a situation.
	iii. analyze the variables in your situation and indicate which variable is dependent.
102.e: Analyze and solve linear equations and pairs of simultaneous linear equations to draw conclusions.	i. find the solution to a system of two linear equations in two variables.
	ii. explain whether a system of two linear equations will have one solution, no solution, or an infinite number of solutions.
	iii. create a system of two linear equations in two variables to model a real-world situation.
	iv. explain the meaning of a solution to a system of two linear equations in two variables in terms of the context being modeled.
102.f: Interpret expressions for linear functions in terms of the situation they model.	i. describe the meaning of the parameters for a linear function in terms of the context.

Content and Practice Expectations	Indicators Choose an artifact where you…
102.g: Solve linear equations and inequalities in one variable.	i. find the solution to a linear equation or inequality in one variable.
	ii. justify a solution to a linear equation or inequality in one variable using logical reasoning or an argument about reasonableness.
102.h: Create equations that describe linear relationships.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate a linear relationship.
	ii. build an equation that models the linear relationship between two quantities in a situation.
102.i: Understand the relevance of modeling with linear functions.	i. see something that surprised you or that you learned about the topic through linear modeling of data. Share what you found surprising or what you learned.
	ii. highlight something you are most proud of in your learning about linear modeling.
102.j: Use a linear function model to determine values of interest in a real-world problem.	i. determine one or more output values for specific inputs to the function.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	MT
102.a.i	X				X					x
102.b.i			X		X					
102.b.ii			X		X					
102.b.iii			X	X	X		X			
102.b.iv			X	X	X		X			

	L1	L2	L3	L4	L5	L6	L7	L8	L9	MT
102.c.i			X	X			X			
102.c.ii			X	X			X			
102.d.i			X							
102.d.ii			X				X			
102.d.iii										
102.e.i						X	X			X
102.e.ii						X	X			
102.e.iii						X				X
102.e.iv						X				X
102.f.i	X	X				X				X
102.g.i								X	X	
102.g.ii		X						X	X	
102.h.i	X	X	X							X
102.h.ii	X	X	X							X
102.i.i	▲	▲				▲				X
102.i.ii	▲	▲				▲				X
102.i.iii	▲	▲				▲				X

102.j.i										X

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.1	K	102.a.i 102.f.i 102.h.i 102.h.ii 102.i.i 102.i.ii 102.i.iii	LP1, LP3, LP6	**Context:** Planning a pizza party **Math focus:** Using equations to represent situations
2.8	K	102.f.i 102.g.ii 102.h.i 102.h.ii 102.i.i 102.i.ii 102.i.iii	LP1, LP3, LP6	**Context:** Parade clean up, filling/emptying tanks **Math focus:** Rearrange equations to solve for variable of interest

2.10	M Amplify Activity 10.1 and Activity 10.2. Consider introducing the concept of domain and asking about values that are not shown on the graph. Consider having students choose a point	102.b.i 102.b.ii 102.b.iii 102.b.iv 102.c.i 102.c.ii 102.d.i 102.d.ii 102.h.i	LP3, LP5, LP6	**Context:** Spending money on games and rides at a carnival, number of nickels and dimes **Math focus:** Connecting equations with graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	on the graph and explain why they chose the point based on the context. Skip Activity 10.3 to give time for exploring suggested ideas.	102.h.ii		
2.11	M Focus on Activity 11.2 and connections between the structure of two-variable linear equations, their graphs, and the situations they represent. To better align to CPEs listed, consider asking students about the domain and prompting students to interpret the rate of change in terms of the contexts.Consider having students choose a point on the graph and explain the meaning of that point in terms of the context. Skip Activity 11.1 and	102.b.iii 102.b.iv 102.c.i 102.c.ii	LP3, LP5, LP6	**Context:** Gallons of water in a tank, how much money is spent on pounds of almonds and dried figs **Math focus:** Connecting equations to graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	Activity 11.3 to give time for exploring suggested ideas.			
3.4 Add Unit 3 Lesson 4 Activities as described	M - Two days. Please note this lesson is also included in Badge M112. Note that the following is based on using that lesson as an introduction to the messiness mathematical modeling can involve. Mastery of the topics introduced in that lesson are not expected at this stage. Please focus on the CPEs listed to guide discussion. Day 1: Activity 4.1 and Activity 4.2. Day 2: Activity 4.3 andActivity 4.4. For Activity 4.4, consider	102.a.i 102.b.i 102.b.ii 102.b.iii 102.b.iv	LP3, LP6	**Context:** Oranges on a scale, ingredient cost vs. sales price, age vs. reaction time, number of bananas vs. money, room size vs. cost to install flooring, temperature vs. volume **Math focus:** Linear models

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	having students create a problem/situation based on one of the graphs and interpret the graph based on the context.			
2.12	M - Two days. Please note this lesson is also included in Badge M101. Consider having students engage in this through a modeling lens as described below, then return to it on Day 2 to attend to the Badge M101 CPEs. Day 1: 12.1-12.2 12.2. Consider focusing on CPE 102.e.ii by having students question how many solutions there could be in the situation. Day 2: 12.3 Assign one graph per	102.e.i 102.e.ii 102.e.iii 102.e.iv 102.f.i 102.i.i 102.i.ii 102.i.iii	LP1, LP3, LP4, LP6, LP7	**Context:** Trail mix, dining halls, purchasing family tickets to a magic show, buying posters **Math focus:** System of equations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	team. Consider focusing on CPE 102.e.iv by having students explain what the solution means in terms of the context.			
2.17	M Please note Activities 17.1, 17.3, and 17.4 from this lesson are best suited for Badge M101. Use Activity 17.2 and utilize recommended Math Language Routines as well as Co-Crafting Questions in the launch of this activity to attend to aspects of mathematical modeling. To better align to CPEs listed, consider asking students about the domain and prompting students to interpret the rate of change in terms of the contexts. Consider having students choose	102.b.iii 102.b.iv 102.c.i 102.c.ii 102.d.ii 102.e.i 102.e.ii	LP2, LP3, LP7	**Context:** Pool passes and gym memberships **Math focus:** Number of solutions for systems of equations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	a point on the graph and explain the meaning of that point in terms of the context.			
2.18	M 18.2: Encourage students to come up with the inequalities based on the context, rather than giving them the inequalities.	102.g.i 102.g.ii	LP1, LP2, LP3, LP5	**Context:** Planning a senior ball, constraints for an elevator **Math focus:** Representing situations in linear inequalities
2.19	M For Activity 19.2: Launch the activity by Co-Crafting Questions Have students justify which orchard they would choose based on the inequalities.	102.g.i 102.g.ii	LP1, LP3, LP5, LP6	**Context:** Class field trip to an orchard, job to pay for tuition **Math focus:** Solutions to linear inequalities in one-variable

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task- A New Heating System	M Consider ending the modeling task by having students reflect on the modeling process—highlight what they learned, what they are most proud of, and how modeling can be used to answer important questions.	102.a.i 102.e.i 102.e.iii 102.e.iv 102.f.i 102.h.i 102.h.ii 102.i.i 102.ii 102.i.iii 102.j.i	LP1, LP3, LP4, LP5, LP6, LP7	**Context:** Replacing an old heating system **Math focus:** Graphing system of equations

M103: Modeling with Functions of Quadratic Type

Illustrative Mathematics–Algebra 1 Unit 6: Introduction to Quadratic Functions

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- The modeling cycle is not well represented, and there are few opportunities for students to identify a problem, formulate their own questions, or report. The use of Math Language Routine 5: Co-Craft Questions may be useful in building these opportunities into some of the existing modeling tasks within the unit. This is necessary to fully meet Content and Practice Expectation 103.a.
- There are minimal opportunities to describe the domain of the function (Indicator 103.b.iii) built into the lessons. Consider explicitly asking students to reflect on the given context or situation and to identify a reasonable domain. Early in the unit, consider building conceptual understanding of domain before naming what students are identifying as finding the domain. For example, you may ask, "*What are some reasonable values for the input? What are reasonable values for the output?"* or " *Would the values (x, y) be a reasonable choice in this situation? Why or why not?",* where the values for x and y do satisfy the equation, but extend beyond the domain. Spaces where we see an opportunity for this are indicated by a star ★.

- The educator will need to bring in space to attend to Indicator 103.h. Consider using this template to offer students space to reflect and document their thinking at key points in time. Spaces where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
103.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
103.b: Interpret quadratic functions that arise in applications in terms of the context.	i. select two different data points that illustrate something interesting or important about the context you are modeling and describe what these points tell you about the topic.
	ii. use a model to compute output values and then interpret the values in the problem's context.
	iii. describe the domain of the quadratic function, including anything noticed about values that are not part of the domain or why the shape of the model might change for particular values of the domain.

Content and Practice Expectations	Indicators Choose an artifact where you…
	iv. describe the contextual meaning of the average rate of change between two points for a quadratic function.
103.c: Analyze quadratic functions using different representations.	i. create a quadratic graph to show a particular context and describe important values that help understand something about the problem you are exploring.
	ii. explain how the parameters of a quadratic model relate to its graph.
103.d: Build quadratic functions that model relationships between two quantities.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate a quadratic relationship.
	ii. build a function that models the quadratic relationship between two quantities in a situation.
103.e: Construct and compare linear and quadratic models and solve problems.	i. distinguish between situations that can be modeled with linear functions and those that can be modeled with quadratic functions, based on the quantities involved or through analysis of the rate of change.
	ii. provide insight into why some relationships are linear and others are quadratic, or explain why, in terms of a situation being modeled, a quadratic function is a better model than a linear function.
103.f: Interpret expressions for quadratic functions in terms of the situation they model.	i. describe the meaning of the parameters for a quadratic function in terms of the context.
103.g: Summarize, represent, and interpret data on two quantitative variables for linear and quadratic model fits.	i. choose a particular representation of your data to communicate something important to a particular person or audience and explain why you made that choice. In this badge, students are encouraged to investigate patterns of association in bivariate data, which includes informal description of the fit of the curve and addresses the usefulness of the model for the particular context.

Content and Practice Expectations	Indicators Choose an artifact where you…
103.h: Understand the relevance of modeling with quadratic functions.	i. recognize something that surprised you or that you learned about the topic through quadratic modeling of data. Share what surprised you or what you learned.
	ii. highlight something you are most proud of in your learning about quadratic modeling.
	iii. explain how modeling can be used to answer important questions.
103.i: Use a quadratic function model to determine values of interest in a real-world problem.	i. determine one or more output values for specific inputs to the function.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	MT
103.a.i																			X
103.b.i	X					X	X			X				X				X	X
103.b.ii	X					X	X							X				X	X
103.b.iii	★				★	X	X			★				★			X	X	
103.b.iv					X	X	X											X	
103.c.i		X			X	X	X							X			X	X	X
103.c.ii		X			X	X	X			X				X			X		X
103.d.i																			X

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	MT
103.d.ii		X	X		X	X	X										X		X
103.e.i		X				X													
103.e.ii		X				X													
103.f.i					X	X	X							X			X	X	X
103.g.i																			
103.h.i	X				▲	▲	▲			▲				▲			▲	▲	▲
103.h.ii	▲				▲	▲	▲			▲				▲			▲	▲	▲
103.h.iii	▲				▲	▲	▲			▲				▲			▲	▲	▲
103.i.i	X				X	X	X							X			X	X	X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.1	M Consider asking students, *What are some reasonable values for the length and width?* to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.i, ii, iii 103.h.i 103.i.i	LP3, LP6, LP7	**Context:** Enclosing a rectangular garden producing the largest possible area **Math focus:** Linear, not linear, length, width, area, plotting points
6.2	K	103.c.i, ii 103.d.ii 103.e.i, ii	LP1, LP2, LP3, LP7	**Context:** Providing sequences of dots and boxes (visual patterns) and explaining how they change **Math focus:** Linear change, quadratic change, patterns, graphs, sequence, tables
6.3	K	103.d. ii	LP1, LP2, LP3	**Context:** Visual patterns **Math focus:** Quadratic patterns and equations
6.4	D			**Context:** Patterns

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	The CPEs do not ask students to compare quadratic and exponential functions.			**Math focus:** Quadratic and exponential patterns
6.5	M Consider asking students, *What are some reasonable values for time?* to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.iii 103.c.i, ii 103.d.ii 103.f.i 103.i.i 103.h.i, ii, iii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Rock dropping from building **Math focus:** Quadratic table, equation, and graph
6.6	M Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.i, ii, iii 103.c.i, ii 103.d.ii 103.e.i, ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Shooting a cannonball into the air **Math focus:** Tables, graphs and equations, distance, time, height, linear and quadratic expressions, vertex, zero, domain, parabola
6.7	M Consider using the linked template (▲) to	103.b.i, ii, iii 103.c.i, ii 103.d. ii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Downloading a movie, business, revenue, area and perimeter of rectangle, patterns,

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	give students access to all indicators of 103.h.	103.f.i 103.h.i, ii, ii 103.i.i		fallen object **Math focus:** Domain, vertex, zeroes, x-intercept
6.8	D No related Content and Practice Expectations. Compares equivalent expressions using diagrams—no context.			**Context:** Purely mathematical **Math focus:** Equivalent quadratic expressions using diagrams
6.9	D No related Content and Practice Expectations. In this lesson, students use rectangular diagrams to reason about the product of two differences or of a sum and difference and to write equivalent expressions.			**Context:** Purely mathematical **Math focus:** Standard form and factored form
6.10	M Consider asking students, *What are some*	103.b.i, iii 103.c.ii 103.h.i, ii, ii	LP1, LP2, LP3, LP7	**Context:** Different forms of an equation with graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	reasonable values for t in this equation? to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.			**Math focus:** Vertical and horizontal intercepts, connecting expressions with graphs
6.11	D Note that the context in this lesson is purely mathematical, making the connection to modeling with geometry weak.			**Context:** Purely mathematical **Math focus:** Graphing from factored form
6.12	D No related Content and Practice Expectations. Students explore the relationship between the coefficient of the squared term and the constant term.			**Context:** Purely mathematical **Math focus:** Identifying equivalent quadratic expressions in standard and factored form and their corresponding graph

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.13	D Note that the context in this lesson is purely mathematical, making the connection to modeling with geometry weak.			**Context:** Purely mathematical **Math focus:** Standard and factored equations, writing equations from graphs
6.14	M Consider asking students, *What are some reasonable values for t in this equation?* to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.i, ii, iii 103.c.i, ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP6, LP7	**Context:** Frog's jump, catapulting pumpkins, baseball being hit by a player, toy rocket, tennis balls **Math focus:** Motion of projectiles, maximum height, vertex, vertical and horizontal intercepts, initial height, zeros
6.15	D Note that the context in this lesson is purely mathematical, making the connection to modeling with geometry weak. Students do have the opportunity to			**Context:** Purely mathematical **Math focus:** Vertex form, standard form, factored form

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	analyze quadratic functions using different representations.			
6.16	D No related Content and Practice Expectations. Students think about how the structure of the equation helps to show whether the vertex of a graph represents the minimum or the maximum value of the function.			**Context:** Purely mathematical **Math focus:** Minimum, maximum, vertex, vertex form, connections to graph of quadratic function
6.17	M Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.iii 103.c.i, ii 103.d.ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Computer animation, peanut jumping a wall, smiley face **Math focus:** Affects to the vertex
7.24	M Engage students in Activity 24. 2 from Unit 7. Allow for use of	103.b.i 103.b.ii 103.b.iii 103.b.iv	LP7	**Context:** Height of a diver above water **Math focus:** Using a given equation to

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	technology. Add questions during discussion to get at 103.b.iii, 103.b.iv, and other indicators listed that are not already built in.	103.c.i 103.f.i 103.h.i 103.h.ii 103.h.iii 103.i.i		gain insights into points of interest about a context
Modeling Task- Designing a Fountain	M Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.a.i 103.b.i, ii 103.c.i, ii 103.d.ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP4, LP5, LP6, LP7	**Context:** Decorative pool with fountain **Math focus:** Quadratic equations in vertex form, vertex, and height

M104: Modeling with Exponential Functions

Illustrative Mathematics–Algebra 1 Unit 5

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- While most of the modeling cycle iss well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of Math Language Routine 5: Co-Craft Questions may be useful in building these opportunities into some of the existing modeling tasks within the unit, particularly some of the more complex tasks toward the end of the unit. This is necessary to fully meet Content and Practice Expectation 104a.
- The educator will need to bring in space to attend to Indicator 104.h. Consider using this template to offer students space to reflect and document their thinking at key points in time. Spaces where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental

challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
104.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
104.b: Interpret exponential functions that arise in applications in terms of the context.	i. select two different data points that illustrate something interesting or important about the context you are modeling and describe what these points tell you about the topic.
	ii. use a model to compute output values and then interpret the values in the context of the problem.
	iii. describe the domain of the exponential function, including anything noticed about values that are not part of the domain or why the shape of the model might change for particular values of the domain.
	iv. describe the contextual meaning of the average rate of change between two points for an exponential function.
104.c: Analyze exponential functions using different representations.	i. create an exponential graph to show a particular context and describe important values that help to understand something about the problem you are exploring.

Content and Practice Expectations	Indicators Choose an artifact where you…
	ii. explain how the parameters of an exponential model relate to its graph.
104.d: Build an exponential function that models a relationship between two quantities.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate an exponential relationship.
	ii. build a function that models the exponential relationship between two quantities in a situation.
104. e: Construct and compare linear and exponential models, solve problems, and draw conclusions.	i. distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions.
	ii. provide insight into why some relationships are linear and others are exponential or explain why, in terms of a situation being modeled, an exponential function is a better model than a linear function.
104.f: Interpret expressions for exponential functions in terms of the situation they model.	i. describe the meaning of the parameters for an exponential function in terms of the context.
104.g: Summarize, represent, and interpret data on two quantitative variables for linear and exponential model fits.	i. choose a particular representation of your data to communicate something important to a particular person or audience and explain why you made that choice. In this badge, students are encouraged to investigate patterns of association in bivariate data, which includes the informal description of the fit of the curve and addresses the usefulness of the model for the particular context.
104.h: Understand the relevance of modeling with exponential functions.	i. recognize something that surprised you or that you learned about the topic through exponential modeling of data. Share what surprised you or what you learned.
	ii. highlight something you are most proud of in your learning about exponential modeling.
	iii. explain how modeling can be used to answer important questions.
104.i. Use functions of exponential type to	i. determine one or more output values for specific inputs to the function.

Content and Practice Expectations	Indicators Choose an artifact where you…
determine values of interest in a real-world problem.	

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	MT
104a.i																						x
104.b.i		x	x	x	x	x	x		x	x		x	x		x							
104.b.ii		x	x	x	x	x	x	x	x	x		x	x		x	x	x		x		x	x
104.b.iii					x	x	x		x													x
104.b.iv										x		x			x							
104.c.i			x		x	x	x	x	x			x			x				x		x	
104.c.ii			x	x	x	x	x	x	x	x		x			x				x		x	
104.d.i									x													
104.d.ii				x			x	x	x				x		x	x	x		x		x	x
104.e.i		x		x	x															x	x	x
104.e.ii		x	x	x	x										x	x			x	x	x	x
104.f.i			x		x	x	x		x	x		x			x		x		x			
104.g.i							x		x							x					x	x

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	MT
104.h.i					▲	▲	▲	▲	▲	▲		▲	▲		▲	▲	▲		▲		▲	▲
104.h.ii					▲	▲	▲	▲	▲	▲		▲	▲		▲	▲	▲		▲		▲	▲
104.h.iii					▲	▲	▲	x	▲	▲		▲	▲		▲	x	x		x		x	x
104.i.i			x	x	x	x	x	x	x	x		x	x			x			x		x	x

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.1	D Not needed to enter Lesson 5.2. Genie context extension not aligned with LPs.			**Context:** Bacteria growth, genie wishes **Math focus:** Linear and non-linear growth basics, review of grades 6,7,8
5.2	M Prioritize Activity 5.2.3 to give students an opportunity to see math as relevant (LP6), especially if students express an interest in	104.b.i 104.b.ii 104.e.i 104.e.ii	LP3, LP6	**Context:** Social media **Math Focus:** Understanding the difference between exponential and linear

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	use of social media. Doing so may mean skipping Activity 5.2.2.			
5.3	K	104.bi 104.b.ii 104.c.i 104.c.ii 104.e.ii 104.f.i 104.i.i	LP3	**Context:** Bacteria growth **Math Focus:** Multiple ways to represent exponential relationships
5.4	M Prioritize activity 5.4.3, and build ideas from activity 5.4.2 into discussion to optimize space for LPs.	104.bi 104.b.ii 104.c.ii 104.d.ii 104.e.i 104.e.ii 104.i.	LP2, LP3, LP6, LP7	**Context:** Value of car → intro decay **Math Focus:** Growth factor, tables, expressions, graphs
5.5	K	104.b.i 104.b.ii 104.b.iii 104.c.i 104.c.ii 104.e.i	LP3, LP4, LP5, LP6, LP7	**Contexts:** Containment of algae bloom, insulin in body **Math Focus:** Continues thread of exponential decay, engages students in aspects of mathematical modeling

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		104.g.i 104.h.iii 104.i.i		represent functions
5.9	M Prioritize activities 5.9.2 and 5.9.3. MLR4: Info Gap activity key to some of the CPEs and LPs.	104.b.i 104.b.ii 104.b.iii 104.c.i 104.c.ii 104.d.i 104.d.ii 104.f.i 104.g.i 104.i.i	LP2, LP3, LP6, LP7	**Context:** Solar costs (decay , paper folding,smartphone sales **Math Focus:** Interpreting and reasoning about exponential functions
5.10	K	104.b.i 104.b.ii 104.b.iv 104.c.ii 104.f.i 104.h.iii 104.i.i	LP1, LP3, LP6	**Context:** Coffee shop, follow up on solar cost **Math Focus:** Review of constant rate of change to compare to average rate; calculate avg rates (including decay) in graph, table, and equation
5.11	D Not needed to enter the next lesson. Optional activity requires			**Context:** Bounce height of balls **Math Focus:** Mathematical modeling

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	materials (types of sports balls), yet the motivation/reason to model ball bounce is not connected to real relevance.			
5.12	K	104.b.i 104.b.ii 104.b.iv 104.c.i 104.c.ii 104.f.i 104.i.i	LP3, LP7	**Context:** Bank accounts **Math Focus:** Exponential decay, exponential equations to graphs
5.13	K	104.b.i 104.b.ii 104.d.ii 104.i.i	LP1, LP3, LP6	**Context:** Value of a computer, mold growth **Math Focus:** Exponential graphs to equations
5.14	D Not needed to enter next lesson, listed as “optional” by authors.			**Context:** Costs of scooters, bikes, etc., taxes and sales **Math Focus:** Review of grade 7 concept of percent increase

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.15	K	104.b.i 104.b.ii 104.b.iv 104.c.i 104.c.ii 104.d.ii 104.e.i 104.f.i	LP1, LP2, LP3, LP6	**Context:** Interest on borrowed amounts **Math Focus:** Using exponential expressions to model repeated interest calculations
5.16	K	104.b.ii 104.d.ii 104.e.ii 104.g.i 104.h.iii 104.i.i	LP1, LP2, LP3	**Context:** Resizing images, earning interest **Math Focus:** Distinguishes between successive percent changes over time versus one overall percent change
5.17	M Prioritize activity 5.17.2.	104.b.ii 104.d.i 104.f.i 104.h.iii	LP3, LP6	**Context:** Interest on earnings, how credit cards work, financial literacy **Math Focus:** Compounding interests (compounding intervals)
5.18	D Not aligned to expectations of this badge and not needed			**Context:** Population growth, interest on earnings **Math Focus:** Properties of exponents

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	for coherence.			
5.19	K The structure of this lesson aids in developing a way to approach open-ended prompts, similar to most open versions of the modeling prompts offered.	104.b.ii 104.c.i 104.c.ii 104.d.ii 104.e.ii 104.h.iii 104.i.i	LP1, LP3, LP6, LP7	**Context:** Compound vs. simple interest **Math Focus:** Comparing growth of linear and exponential functions
5.20	K	104.e.i 104.e.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math Focus:** Rates of change, what makes something linear vs exponential
5.21	K	104.b.ii 104.c.i 104.c.ii 104.d.ii 104.e.i 104.e.ii 104.g.i 104.h.iii 104.i.i	LP1, LP2, LP3, LP6, LP7	**Context:** Population [cities → world] **Math Focus**: Using linear and exponential functions for modeling

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task- Critically Examining National Debt	K Two versions are given: one in which data is provided, and one in which other students do research to find data to use.	104.a.i 104.b.ii 104.b.iii 104.d.ii 104.e.i 104.e.ii 104.g.i 104.h.iii 104.i.i	LP1, LP2, LP3, LP6, LP7	**Context:** US national debt **Math Focus**: Modeling with linear and exponential functions

M111 Modeling with Data: One-Variable Measurement Data

Illustrative Mathematics–Algebra 1 Unit 1: One-variable Statistics **Illustrative Mathematics- Algebra 2 Unit 7: Statistical Inferences**

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be accomplished by offering opportunities for students to reflect on how the knowledge and skills they're developing are being used to impact decision making at the local level (classroom, campus, or neighborhood). This can be done in conjunction with attention to Indicator 111g.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of Math Language Routine 5: Co-Craft Questions may be useful in building these opportunities into some of the existing modeling tasks within the unit, particularly some of the more complex tasks toward the end of the unit. This is necessary to fully meet Content and Practice Expectation 111.a.
- Educators will need to bring in space to attend to Indicator 111.g by using selected lessons from IM Algebra 2 Unit 7-Statistical Inference. Spaces where we see an opportunity for this are indicated by a square ■ in the unit overview chart.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental

challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
111.a: Engage in the modeling cycle.	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
111.b: Develop statistical questions in the course of modeling with one-variable measurement data.	i. develop a statistical question to investigate.
	ii. explain or justify why your question is statistical.
111.c: Interpret differences in shape, center, and spread in the context of data sets, accounting for the possible effects of extreme data points (outliers).	i. determine and construct an appropriate graphical representation of collected data, such as box plot, line plot, histogram, or other data display.
	ii. describe the shape, center, and spread of a data set.
	iii. interpret the real-world meaning of the shape, center, and spread of a data set.

	iv. identify and explain the effect of an outlier on the shape, center, and spread of a data set.
111.d: Use measures of center and variability to describe one-variable measurement data.	i. find the mean or median of a set of data and explain its meaning.
	ii. find the interquartile range or standard deviation of a set of data and explain its meaning within the context of the data.
111.e: Generate a random sample in the course of modeling with one-variable measurement data.	i. describe a process for generating a random sample to investigate a statistical question.
	ii. explain why a proposed process generates a random sample that is appropriate for the statistical question being investigated.
111.f: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	i. construct a box plot, line plot, histogram, or other data display for each of two related data sets.
	ii. find measures of center (mean or median) and variability (standard deviation or interquartile range) for two data sets.
	iii. compare two data sets in terms of their center and variability.
	iv. explain the choice of measures of center and variability used to compare the two populations.
111.g: Estimate population percentages by using a normal curve to approximate a data distribution.	i. explain why a normal distribution is an appropriate approximation for a data set.
	ii. find an estimated population percentage by using a normal curve to approximate a data distribution.
	iii. explain the meaning of an estimated population percentage.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	MT
111.a.i																X				X
111.b.i	X															X				X
111.b.ii	X															X				X
111.c.i		X	X						X	X						X				X
111.c.ii		X	X	X						X	X									X
111.c.iii			X	X							X									X
111.c.iv									X	X		X		X						X
111.d.i		X	X		X					X	X	X		X						
111.d.ii		X	X		X						X	X		X						
111.e.i																				
111.e.ii																				
111.f.i																				X
111.f.ii											X		X							
111.f.iii											X		X		X					X
111.f.iv											X		X		X					X
111.g.i																	■	■	■	■

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	MT
111.g.ii																		■	■	■
111.g.iii																		■	■	■

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
1.1	K Needed for later activity.	111.b.i 111.b.ii	LP2, LP4	**Context:** Last names, birthdays, mileage from home, food, movies **Math focus:** Writing and collecting Statistical and non-statistical questions and numerical and categorical data
1.2	K Practice with data displays for Lesson 3. Consider asking students to think about why anyone would want to know the number of days it takes for a tomato plant to start producing fruit as a launch to the activities in this lesson to attend to LP6.	111.c.i 111.c.ii 111.d.i 111.d.ii	LP1, LP3, LP6	**Context:** Tomato plants **Math focus:** Data represented by dot plots, histograms, and box plots
1.3	K Data display for Lesson 1.	111.c.i 111.c.ii 111.c.iii 111.d.i	LP1, LP3	**Context:** Last names, birthdays, mileage from home, food, movies **Math focus:** Creating two different data

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		111.d.ii		displays from information collected in Lesson 1
1.4	K	111.c.ii 111.c.iii	LP1, LP2, LP3, LP6	**Context:** Games, temperatures, restaurants, vocabulary, trees, weight of crystals, core samples **Math focus:** Describing the shape of distributions using formal language and inventing contexts for distributions with different shapes
1.5	K	111.d.i 111.d.ii	LP1, LP6	**Context:** Heart rates **Math focus:** Calculating MAD and IQR
1.6	D Delete unless students do not have any experience with spreadsheets.		LP7	**Context:** Variety of numbers and formulas **Math focus:** Creating formulas in a spreadsheet
1.7	D Delete unless students do not have any experience with spreadsheets.		LP7	**Context:** Speed limit, right triangles, recipes **Math focus:** Creating formulas in a spreadsheet

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
1.8	D Delete unless students do not have any experience with spreadsheets.		LP7	**Context:** Birthdays, gym memberships **Math focus:** Entering values and patterns into a spreadsheet
1.9	M Change to include statistical data that actually connects to real life contexts, authentic challenges, and/or interests for students.	111.c.i 111.c.iv	LP3, LP5, LP6, LP7	**Context:** Statistical data **Math focus:** Display and investigate data using technology
1.10	K	111.c.i 111.c.ii 111.c.iv 111.d.i	LP3, LP7	**Context:** Battle Royale, darts, dartboard **Math focus:** Measures of center and the shape of distribution
1.11	K	111.c.ii 111.c.iii 111.d.i 111.d.ii 111.f.ii 111.f.iii 111.f.iv	LP1, LP2, LP3	**Context:** Miles driven in a month, sea scallop shells **Math focus:** Variability using data

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
1.12	K	111.c.iv 111.d.i 111.d.ii	LP2, LP7	**Context:** Purely mathematical **Math focus:** Standard deviation as a measure of variability
1.13	K	111.f.ii 111.f.iii 111.f.iv	LP1, LP2, LP3, LP6	**Context:** African and Asian elephants, trees, forest, German language, raisins in cereal, granny smith apples, navel oranges, points scored in a basketball game **Math focus:** Standard deviation in context
1.14	K	111.c.iv 111.d.i 111.d.ii	LP2, LP5, LP6, LP7	**Context:** Healthcare spending, number of crimes reported in CA colleges, number cube, siblings, and biodiesel mass **Math focus:** The impact outliers have on measures of center and variability
1.15	K	111.f.iii 111.f.iv	LP3, LP6	**Context:** Bowling, marathon times, political podcast, cooking podcast, stress testing concrete, wide receiver times, tight end times

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
				Math focus: Comparing data sets using variability (standard deviation) and measures of center
1.16	K	111.a.i 111.b.i 111.b.ii 111.c.i 111.e.i 111.e.ii	LP1, LP2, LP5, LP6, LP7	**Context:** Dropping a ruler, heights and handedness **Math focus:** Data collection, data analysis, answering a statistical questions
A2.7.5	M Can amplify relevance by inviting students to explore why hand size might matter for piano players in Activity 5.2. In doing so, look to bring **relative frequency histogram** questions from Activity 5.3 into the discussion of 5.2.	111.g.i	LP3, LP5, LP6, LP7	**Context:** Hand span and playing piano, distribution of butter sticks manufacturing **Math focus:** Introduction of normal distribution, relative frequency histogram, and normal curve
A2.7.6	M Add this lesson from A2 Unit 7. Consider asking students to reflect on	111.g.i 111.g.ii 111.g.iii	LP3, LP7	**Context:** Length of short stories, website load times **Math focus:** Making connections

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	the following question to engage students in LP6: *why do you think understanding these distributions is important?*			between histograms, normal distributions, the mean, and the standard deviation to estimate population proportions
A2.7.7	M Add this lesson from A2 Unit 7. Consider asking students to reflect on the following question to engage students in LP6: *why do you think understanding these distributions is important?*	111.g.i 111.g.ii 111.g.iii	LP3, LP7	**Context:** Lifespan of lightbulbs, wait times at a restaurant **Math focus:** Finding area under a normal curve and interpreting the area as a proportion of values in context
Modeling Task 1- Display Your Data	M Broaden the scope of this prompt to give students the option to apply their understanding of normal distribution.	111.a.i 111.b.i 111.b.ii 111.c.i 111.c.ii 111.c.iii 111.c.iv 111.f.i 111.f.iii	LP4, LP6, LP7	**Context:** Student choice **Math focus:** Statistical question, data collection, comparing data sets, using measure of center and variability, data display, outliers, shape of distribution

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		111.f.iv		

M112: Modeling with Data- Two-Variable Measurement Data

Illustrative Mathematics–Algebra 1 Unit 3: Two-variable Statistics

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- As the third unit, this IM is designed to follow work on linear functions. As a result, a large majority of the tasks focus on bivariate numerical data that is linear in nature. Because of this structure, it is expected that prompts early in a student's journey through this unit give students opportunities to attend to CPE 112.d. As work in the unit progresses, students should gain comfort and proficiency in using tools to fit a model to two-variable measurement data, CPE 112.e. On occasion, students will have opportunities to recognize nonlinear trends. Some examples have been found to look exponential in nature. From work in this unit, it is sufficient for students to name such trends as nonlinear to meet the expectations of CPE 112.e. It is encouraged that students explore regression tools using technology available to determine the choice of linear or nonlinear function to model two-variable measurement data.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of Math Language Routine 5: Co-Craft Questions may be useful in building these opportunities into some of the existing modeling tasks within the unit,

particularly some of the more complex tasks toward the end of the unit. This is necessary to fully meet Content and Practice Expectation 112a.

- Educators will need to bring in space to attend to Indicator 112.f.iii and Indicator 112.h.ii. Consider using this template to offer students space to reflect and document their thinking at key points in time. Places where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see

themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
112.a: Engage in the modeling cycle.	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
112.b: Use scatter plots to represent two-variable measurement data.	i. construct (by hand or using technology) a scatter plot from collected data.
	ii. explain the meaning of a data point plotted on a scatter plot.
112.c: Describe visible patterns between two data sets in a scatter plot.	i. identify the type of association visible in a scatter plot.
	ii. give a hypothesis that might explain the association identified in a scatter plot.

Content and Practice Expectations	Indicators Choose an artifact where you…
112.d: Informally fit a straight line for scatter plots that suggest a linear association.	i. sketch a line that follows a pattern of linear association.
	ii. justify location and direction of a line fitted to data.
112.e: Fit a linear, quadratic, or exponential function to two-variable measurement data.	i. graph a linear, quadratic, or exponential function that models two-variable measurement data.
	ii. find the equation for a linear, quadratic, or exponential function that models two-variable measurement data.
	iii. justify the choice of linear, quadratic, or exponential function to model two-variable measurement data.
112.f: Use functions that have been fitted to two-variable measurement data to answer questions about the relationship being modeled.	i. explain the meaning of parameters for a function fitted to two-variable measurement data.
	ii. use a function fitted to two-variable measurement data to predict values.
	iii. describe any limitations to the model relative to answering particular questions of interest.
112.g: Assess the fit of a function using the correlation coefficient, residuals, and other tools.	i. find the correlation coefficient for a function fitted to two-variable measurement data.
	ii. explain the meaning of a correlation coefficient in the context of the problem.
	iii. plot residuals for a function fitted to two-variable measurement data.
	iv. interpret a residual plot in terms of a function's goodness of fit.
112.h: Explain insights gained from analyzing two-variable measurement data and limitations of curves fitted to data.	i. develop a proposal or recommendation grounded in your analysis.
	ii. explain a limitation of a curve fitted to data.
	iii. distinguish between correlation and causation.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	MT
112.a.i										X	X
112.b.i				X	X	X				X	X
112.b.ii				X							X
112.c.i							X	X	X	X	X
112.c.ii								X	X	X	X
112.d.i				X	X					X	
112.d.ii				X	X						
112.e.i				X			X	X		X	X
112.e.ii							X			X	X
112.e.iii							X			X	X
112.f.i				X	X			X	X	X	X
112.f.ii				X	X			X		X	X
112.f.iii								▲	▲	▲	▲
112.g.i							X	X		X	X

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	MT
112.g.ii								X		X	X
112.g.iii						X				X	X
112.g.iv						X				X	X
112.h.i									X	X	X
112.h.ii								▲	▲	▲	▲
112.h.iii									X	X	X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.1	D Focus of the lesson is not within the scope of this badge. Not needed for the next lesson.			**Context:** Office supplies, fruit flies, running a dentist office **Math focus:** Two-way tables
3.2	D Focus of the lesson is not within the scope of this badge. Not needed for next lesson.			**Context:** College degrees, cat population, treatment of colds **Math focus:** Relative frequency tables
3.3	D Focus of the lesson is not within the scope of this badge. Not needed for next lesson.			**Context:** Dessert preference, nitrate levels in coral reef, student choice **Math focus:** Associations in categorical data
3.4	M Please note this lesson is also included in Badge M102. Consider revisiting parts of this lesson, like Activity 4.1 and/or Activity 4.3, to	112.b.i (data is given) 112.b.ii 112.d.i 112.d.ii 112.f.i 112.f.ii	LP2, LP3, LP7	**Context:** Oranges, ingredient cost, sale prices, mass, fuel economy **Math focus:** Creating scatter plot from given data, identifying line of best fit, slope, y-intercept, making predictions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	reintroduce this topic when addressing this Badge.			
3.5	K This is a review of topics from Lesson 4. Consider using this as a way to reintroduce this topic of study if you use Lesson 4 with Badge M102.	112.b.i (data is given) 112.d.i 112.d.ii 112.f.i 112.f.ii	LP2, LP7	**Context:** Weight of ice cream, degrees Celsius **Math focus:** Creating scatter plot from given data, identifying line of best fit, slope, y-intercept
3.6	K	112.b.i (data is given) 112.g.iii 112.g.iv	LP2, LP7	**Context:** Number and weight of oranges **Math focus:** Residuals, fit a function to data
3.7	M Include “Are you ready for more?” activity found in Activity 7.3 to meet CPE 112.e.	112.c.i 112.e.i 112.e.ii 112.e.iii 112.g.i	LP1, LP2, LP3, LP7	**Context:** Card sort, walking, texting **Math focus:** Linear and nonlinear models, correlation coefficient
3.8	K Activity 8.2 lends itself to the use of Form for Aligning Lessons	112.c.i 112.c.ii 112.e.i 112.f.i	LP1, LP2, LP6, LP7	**Context:** Temperature, free throws, student height, waiting room, cereal, distance, time, cars, trips, dolphins

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	Reflection- Limitation of Curve Fitting.	112.f.ii 112.f.iii 112.g.i 112.g.ii 112.h.ii		**Math focus:** Type of association, strength of correlation, using an equation to make predictions, interpreting the coefficients, describing variables that influence the situation, slope, y-intercept
3.9	K Activity 9.2 lends itself to the use of Form for Aligning Lessons Reflection- Limitation of Curve Fitting to meet 112.f.iii. Activity 9.3 lends itself to the use of Form for Aligning Lessons Reflection- Limitation of Curve Fitting to meet 112.f.iii and 112.h.iii. Leverage Activity 9.3 to give students access to full scope of CPE 112.h.	112.c.i 112.c.ii 112.f.i 112.f.iii 112.h.i 112.h.ii 112.h.iii	LP2, LP3, LP4, LP6	**Context:** Car (sale price, cup holders, oil changes, miles driven), inches of rain, rain jackets, books, holiday party, height, score of vocabulary, advertisements **Math focus:** Type of relationships between variables, casual relationships, description of how variables are related, correlation, causation
3.10	K Both activities in this lesson lend themselves to the use of Form for	112.a.i 112.b.i 112.c.i 112.c.ii	LP1, LP2, LP3, LP4, LP5, LP6, LP7	**Context:** Anthropologist, fossilized humerus bones, number of penalties a team receives, number of wins

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	Aligning Lessons Reflection- Limitation of Curve Fitting to meet 112.f.iii and 112.h.iii.	112.d.i 112.d.ii 112.e.i 112.e.ii 112.e.iii 112.f.i 112.f.ii 112.f.iii 112.g.i 112.g.ii 112.g.iii 112.g.iv 112.h.i 112.h.ii 112.h.iii		**Math focus:** Collecting, summarizing, interpreting, and drawing conclusions from bivariate data, best fit lines, residuals, correlation coefficients
Modeling Task- College Characteristics	K Students may use qualitative data to analyze, in which case lessons 1-3 of this unit will be key. In these cases, encourage students to include bivariate numerical data, which are visualized using scatter plots and	112.a 112.b 112.c 112.e 112.f 112.g 112.h	LP4, LP6, LP7	**Context:** College characteristics, SAT scores, ownership, acceptance rate, average cost, enrolled students, median earnings 10 years after graduation, highest degree awarded, completion rate, median ACT score **Math focus:** Conjecture, how data are related, presentation of data

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	lines of best fit, in order to get to the Content and Practice Expectations of this Badge.			

M151: Modeling with Geometry

Illustrative Mathematics–Geometry Unit 5: Solid Geometry

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum and opportunities to use the embedded math language routines to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of Math Language Routine 5: Co-Craft Questions may be useful in building these opportunities into some of the existing modeling tasks within the unit.
- Educators will need to bring in space to attend to Content and Practice Expectation 151.c.i. Consider using this modeling prompt to offer students an opportunity to engage in this CPE. Additional CPEs aligned with this task are marked by a triangle (▲).

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental

challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
151.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
151.b: Use geometric shapes, their measures, and their properties to describe objects.	i. represent a context, situation, or object from outside mathematics as a mathematical object.
	ii. explain how the mathematical object(s) represent the real-life context, situation, or object.
	iii. use properties of the mathematical object to determine quantities of interest and interpret those values within the context.
151.c: Apply concepts of density based on area and volume in modeling situations.	i. apply concepts of density based on area in modeling situations.
	ii. apply concepts of density based on volume in modeling situations.
151.d: Apply geometric methods to solve design problems.	i. design an object or structure to satisfy physical constraints or features in a real-life context.

Content and Practice Expectations	Indicators Choose an artifact where you…
151.e: Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	i. use informal justifications to explain or reason about circumference, area, and volume formulas. Justifications may reflect dissection arguments, Cavalieri's principle, and/or informal limit arguments.
151.f: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	i. solve real-life and mathematical problems using volume formulas for cylinders, pyramids, cones, and spheres.
151.g: Solve real-world problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	i. analyze the context of the problem to determine which measurable geometric attributes are important to reasoning about the problem.
	ii. solve real-life problems using area, volume, and surface area of two- and/or three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and/or right prisms.
151.h: Use the Pythagorean theorem to solve right triangles in applied problems.	i. recognize situations where the Pythagorean theorem is a useful tool for representing and solving problems.
	ii. solve applied problems using the Pythagorean theorem.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	MT	MT2
151.a.i																				▲
151.b.i	X	X	X		X		X	X	X	X	X			X		X		X	X	
151.b.ii	X	X	X		X		X	X	X	X	X			X		X		X	X	
151.b.iii			X		X		X	X		X	X		X	X	X	X		X	X	
151.c.i																				▲
151.cii																	X			
151.d.i	X	X	X				X	X						X	X	X			X	
151.e.i	X	X	X	X		X	X		X	X		X	X							
151.f.i									X	X	X		X	X	X	X	X		X	
151.g.i					X		X	X	X	X	X			X		X	X	X	X	▲
151.g.ii					X		X	X			X			X		X	X	X	X	▲
151.h.i											X				X					
151.h.ii											X				X					

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.1	M As students engage in creating and examining 3D images, consider asking students to identify objects they see or have seen in daily life that can be represented using these images to fully attend to expectations of CPE 151.b. Image C in the warm-up gives students an opportunity to learn about “tavaillons” as well as explore the reasons why someone would design a building that has a conical rooftop. Consider engaging students in Activity 1.2: “Are you ready for more?”, as well as inviting students to create an image in the	151.b.i 151.b.ii 151.d.i 151.e.i	LP2, LP6, LP5, LP7	**Context:** Purely mathematical—digital and hand drawn three-dimensional figures **Math focus:** Axis of rotation, solid of rotation, cylinders, cone, sphere

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	same fashion of an object they see in their daily life.			
5.2	M Consider asking students to name things they encounter that Activity 2.1: “Slice this”, may apply in order to deepen engagement with CPE 151.b. Utilize Activity 2.3’s “Are you ready for more?” prompt to get to the full extent of CPE 151.b.	151.b.i, ii 151.d.i 151.e.i	LP1, LP2, LP3, LP6, LP7	**Context:** Coins, three-dimensional slabs, 3D-printer, cheese, toy rocket **Math focus:** Cross sections, sphere, prism, faces, cylinder, pyramid, cone
5.3	K	151.b.i, ii, iii 151.d.i 151.e.i	LP2, LP3, LP5, LP6	**Context:** Purely mathematical—pyramid mobile **Math focus:** Dilation, center of dilation, cross-sections, scale factor
5.4	K Good foundation for engaging in applications grounded in density	151.e.i	LP1, LP2, LP3	**Context:** Purely mathematical—dilated rectangle, dilated blob **Math focus:** Dilating by scale factors for

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	(CPE.151.c.i).			area
5.5	K	151.b.i, ii, iii 151.g.i, ii	LP1, LP2, LP6, LP7	**Context:** Transamerica building, scaling a painting, train station **Math focus:** Scaling, area, cross-sections, pyramid, rate of change, graphing
5.6	K Good foundation for engaging in applications grounded in density (CPE.151.c.ii).	151.e.i	LP1, LP2, LP3, LP5, LP7	**Context:** Sierpinski's triangle **Math focus:** Scaling surface area and volume of cube
5.7	M Consider amplifying Activity 7.3 to give students time to research and make sense of space satellite design. This will amplify connections to LPs listed.	151.b.i, ii, iii 151.d.i 151.e.i 151.g.i, ii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Shipping company, satellite, superhero action figure **Math focus:** Scale, dilation, graphing, equations, surface area, volume, cube root

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.8	M If limited on time, take out Activity 8.2.	151.b.i, ii, iii 151.d.i 151.g.i, ii	LP1, LP2, LP3, LP5, LP6	**Context:** Beverage company, Burj Khalifa, Eiffel Tower, dog food, aquarium **Math focus:** Dilated solids, scale factors, cylinders
5.9	K This lesson builds on Lesson 5.1.	151.b.i, ii 151.e.i 151.f.i 151.g.i	LP1, LP2, LP3	**Context:** Water transfer **Math focus:** Volume of cylinders and prisms, composite figures, hollow cylinder, solids of rotation
5.10	K	151.b.i, ii, iii 151.e.i 151.f.i 151.g.i	LP1, LP2, LP3, LP6, LP7	**Context:** Stack of coins, index cards **Math focus:** Right and oblique cylinders, volume, cross-sections, Cavalieri's principle
5.11	K If students complete this badge prior to M155: Right Triangle Trigonometry, then an additional measurement will need to be given for students to be able to use Pythagorean	151.b.i, ii, iii 151.f.i 151.g.i, ii 151.h.i, ii	LP1, LP2, LP3, LP4, LP6	**Context:** Cayan Tower **Math focus:** Pythagorean theorem, trigonometric ratio, volume

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	theorem to solve for unknown lengths in various prompts.			
5.12	K	151.e.i	LP1, LP2, LP3, LP4	**Context:** Purely mathematical—pyramids, cones, cylinders, prisms **Math focus:** Platonic solids, faces, apex, cross-sections, right and oblique pyramids and other solids
5.13	K This lesson uses the pyramids made in 5.12.	151.b.iii 151.e.i 151.f.i	LP1, LP2, LP3	**Context:** Purely mathematical—using cross-sections to compare volume of prisms, cylinders, and cones **Math focus:** Formula for the volume of pyramids
5.14	M Consider amplifying Activity 14.2's "Are you ready for more?" prompt in lieu of the prompts in Activity 14.2's "Practice with Pyramids" to engage students with	151.b.i, ii, iii 151.d.i 151.f.i 151.g.i, ii	LP1, LP2, LP4, LP6, LP7	**Context:** Pyramid of Giza, ice sculpture **Math focus:** Comparing volume of pyramids, prisms and cones with a focus on pyramids, finding dimensions given volume

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	CPE 151.g.			
5.15	K	151.b.iii 151.d.i 151.f.i 151.h.i, ii	LP1, LP3, LP7	**Context:** Purely mathematical—strategies for selecting formulas and determining necessary dimensions, measuring packet of sunflower seeds **Math focus:** Volume, Pythagorean theorem, trigonometry, composite solids
5.16	K	151.b.i, ii, iii 151.d.i 151.f.i 151.g.i, ii	LP1, LP2, LP6, LP7	**Context:** Elephants' enclosure at a zoo, lithium ion battery, manufacturing packaging, spherical elephants and cylindrical snakes, comparison of human, ant, and elephant **Math focus:** Maximizing area, surface area and volume, raw and relative strength, surface area to volume ratio
5.17	K	151.c.ii 151.f.i 151.g.i, ii	LP1, LP2, LP3, LP4, LP6	**Context:** Volume problem-solving concepts to situations involving density: pillow, anchor, pencils, number of fish in a tank, determining if a lime will float or sink

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
				Math focus: Volume and density, unit conversions
5.18	K This lesson is very similar to Lesson 5.5.	151.b.i, ii, iii 151.g.i, ii	LP1, LP2, LP6, LP7	**Context:** Making balloons for parades with restrictions, determining the amount of helium needed **Math focus:** Unit conversions, scale factor, volume, graphing, equations
Modeling Task- A New Container	M Consider opening the prompt by asking students to define the problem they want to solve as well as giving students the opportunity to identify quantities of interest to get to the full aspects of CPE 151.a. (Doing so may open space for other CPEs, not listed, to be included depending on the figures students choose.)	151.a.i 151.b.i, ii, iii 151.d.i 151.f.i 151.g.i, ii	LP1, LP4, LP5, LP6, LP7	**Context:** Designing a container or image showing what it will look like and what its dimensions will be, including calculations **Math focus:** Designing a three-dimensional figure that can hold 16 fluid ounces

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task 2- Representation in the United States Congress	M Add a prompt like Example 4 from Annotated Examples in M151: Modeling with Geometry.	151.a.i 151.c.i 151.g.i, ii	LP1, LP2, LP3, LP4, LP6, LP7	**Context:** Congress and House of Representatives **Math focus:** Area density based in modeling situations

M152: Reasoning and Proof through Congruence

Illustrative Mathematics–Geometry Unit 2-Congruence

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum and opportunities to use the embedded math language routines to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- Educators will need to bring in space to attend to Indicator 152 a.i. Consider using Geometry Unit 1 Lessons 10, 13-14 to offer students space to use these skills. Spaces where we see an opportunity for this are indicated by a square ⬛.
- Educators will need to bring in space to attend to Indicator 152.j.i and 152.j.ii. Consider using Geometry Unit 1 Lesson 5 to offer students space to use these skills. Spaces where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
152.a: Use definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	i. explain what a rotation, reflection, or translation is using appropriate terms: angle, circle, perpendicular line, parallel line, or line segment.
152.b: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using tools such as graph paper, tracing paper, or geometry software.	i. create a rotated, reflected, and/or translated figure using graph paper, tracing paper, or geometry software.
	ii. explain how you rotated, reflected, and/or translated a figure.
152.c: Specify a sequence of transformations that will carry a given figure onto another.	i. describe a sequence of transformations that would carry a figure onto another figure.
152.d: Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	i. write an argument or explanation for why two figures are congruent, using similarity transformations in your reasoning.
	ii. write an argument or explanation for why two figures are not congruent, using similarity transformations in your reasoning.
152.e: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	i. demonstrate using an example that if two triangles are congruent, then corresponding angles and corresponding sides are also congruent.
	ii. explain why, in general, if two triangles are congruent, then corresponding angles and corresponding sides are also congruent.
	iii. demonstrate using an example that if two triangles have corresponding angles and corresponding sides that are congruent, then the two triangles are also congruent.

Content and Practice Expectations	Indicators Choose an artifact where you…
	iv. explain why, in general, if two triangles have corresponding angles and corresponding sides that are congruent, then the two triangles are also congruent.
152.f: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	i. demonstrate using an example that if two triangles meet one of the triangle congruence criteria (ASA, SAS, or SSS), then they are congruent according to the definition of congruence in terms of rigid motions.
	ii. explain why, in general, if two triangles meet one of the triangle congruence criteria (ASA, SAS, or SSS), then they are congruent according to the definition of congruence in terms of rigid motions.
152.g: Prove theorems about lines and angles.	i. find a missing length or angle measurement in a diagram containing lines and angles and explain how you determined your answer.
	ii. read a proof of a statement about lines and angles (a specific case or in general) and summarize or explain it in your own words.
	iii. write a proof of a statement about lines and angles (a specific case or in general), using rigid transformations or the concept of congruence in your reasoning.
152.h: Prove theorems about triangles.	i. find a missing length or angle measurement in a triangle and explain how you determined your answer.
	ii. read a proof of a statement about a triangle or triangles in general and summarize or explain it in other words.
	iii. write a proof of a statement about a triangle or triangles in general, using rigid transformations or the concept of congruence in your reasoning.
152.i: Prove theorems about parallelograms.	i. find a missing length or angle measurement in a parallelogram and explain how you determined your answer.
	ii. read a proof of a statement about a parallelogram or parallelograms in general and

Content and Practice Expectations	Indicators Choose an artifact where you…
	summarize or explain it in your own words.
	iii. write a proof of a statement about a parallelogram or parallelograms in general, using rigid transformations or the concept of congruence in your reasoning.
152.j: Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	i. construct a geometric figure using compass and straightedge, dynamic geometric software, or other tools.
	ii. explain how you constructed a geometric figure.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	MT
152.a.i	⬛	⬛	⬛													
152.b.i	X	X				X										
152.b.ii					X	X			X							
152.c.i	X	X			X	X			X							
152.d.i	X	X	X		X	X			X							X
152.d.ii		X	X													X
152.e.i	X			X												
152.e.ii	X			X										X		

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	MT
152.e.iii	X	X														
152.e.iv	X	X		X										X		
152.f.i						X	X		X	X						
152.f.ii						X	X		X	X						
152.g.i					X			X								
152.g.ii					X			X						X		
152.g.iii					X			X								
152.h.i						X			X							
152.h.ii						X	X	X	X	X				X		
152.h.iii						X	X	X	X					X		
152.i.i													X			
152.i.ii										X		X	X			
152.i.iii									X			X	X		X	
152.j.i								▲						▲		
152.j.ii								▲						▲		

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.1 Add Unit 1 Lesson 10 Activities as described	M Two days to complete. Recommended sequence of activities: Unit 1 Lesson 10 Activity 1 (1.10.1) and Unit 1 Lesson 10 Activity 2 (1.10.2) to introduce language of transformations, reflections, and rotations. Skip Unit 2 Lesson 1 Activity 1. Do Activity 2.1.2 and Activity 2.1.3	152.a.i 152.b.i 152.c.i 152.d.i 152.e.i 152.e.ii 152.e.iii 152.e.iv	LP1, LP2	**Context:** All mathematical context; no real life context used **Math focus:** Corresponding sides and angles
2.2 Add Unit 1 Lesson 13 Activities as described	M Recommended sequence of activities: Use 1.13.1 and 1.13.2 to focus on students using appropriate language to write a sequence of rotations and create a sequence of rotations. Complete Unit 2 Lesson 2 Activity 1 (2.2.1) and Activity 2.2.2. Skip Activity 2.2.3 if	152.a.i 152.b.i 152.c.i 152.d.i 152.d.ii 152.e.iii 152.e.iv	LP3	**Context:** All mathematical context; no real life context used **Math focus:** Corresponding parts

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	needed.			
2.3 Add Unit 1 Lesson 14 Activities as described	M Recommended sequence of activities: Use 1.14.3 to give students an opportunity to create triangles using rotations. Use 2.3.2 to focus on corresponding parts of triangles. Skip Activities 2.3.1 and 2.3.3 if needed	152.a.i 152.d.i 152.d.ii	LP2, LP3	**Context:** All mathematical context; no real life context used **Math focus:** Congruent triangles
2.4	K	152.e.i 152.e.ii 152.e.iv	LP1, LP2	**Context:** All mathematical context; no real life context used **Math focus:** Congruent triangles
2.5	K As prompts permit, ask students about measures of corresponding sides and angles to meet CPE 152.g.i.	152.b.ii 152.c.i 152.d.i 152.g.i 152.g.ii 152.g.iii	LP2	**Context:** All mathematical context; no real life context used **Math focus:** Congruent segments
2.6	K	152.b.i 152.b.ii 152.c.i 152.d.i	LP2, LP3, LP7	**Context:** All mathematical context; no real life context used **Math focus:** Side-angle-side triangle

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		152.f.i 152.f.ii 152.h.i 152.h.ii 152.h.iii		congruence
2.7	K	152.f.i 152.f.ii 152.h.ii 152.h.iii	LP2, LP3, LP7	**Context:** All mathematical context; no real life context used **Math focus:** Angle-side-angle triangle congruence
2.8 Add Unit 1 Lesson 5 Activities as described	M Recommended sequence of activities: Use 2.8.1 to warm up students. Use 1.5.1 and 1.5.2 to give students an opportunity to construct perpendicular bisectors, followed by 2.8.2 and 2.8.3. As prompts permit, ask students about measures of corresponding sides and angles to meet CPE 152.g.i.	152.g.i 152.g.ii 152.g.iii 152.h.ii 152.h.iii 152.j.i 152.j.ii	LP1, LP2, LP7	**Context:** All mathematical context; no real life context used **Math focus:** Perpendicular bisector theorem

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.9	K	152.b.ii 152.c.i 152.d.i 152.f.i 152.f.ii 152.h.i 152.h.ii 152.h.iii 152.i.iii	LP1, LP3, LP7	**Context:** All mathematical context; no real life context used **Math focus:** Side-side-side triangle congruence, parallelograms
2.10	M Prioritize Activity 10.2 to focus on SAS, ASA, and SSS triangle congruence theorem. Skip Activity 10.3 if needed.	152.f.i 152.f.ii 152.h.ii 152.i.ii	LP2	**Context:** All mathematical context; no real life context used **Math focus:** Proofs, congruence
2.11	D Not needed to enter the next lesson. Lesson marked as optional by authors.			**Context:** All mathematical context; no real life context used **Math focus:** Students study ambiguous case of S-S-A triangle congruence
2.12	K	152.i.ii 152.i.iii	LP2	**Context:** All mathematical context; no real life context used **Math focus:** Parallelogram proofs
2.13	K	152.i.i 152.i.ii	LP1, LP2	**Context:** All mathematical context; no real life context used

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		152.i.iii		**Math focus:** Parallelogram proofs
2.14 Add Unit 1 Lesson 5 Activities as described	M Recommended sequence of activities: Use 1.5.3 to give students an opportunity to create an angle bisector. Follow with 2.14.1 and 2.14.2. Skip 2.14.3 if needed.	152.e.ii 152.e.iv 152.g.ii 152.h.ii 152.h.iii 152.j.i 152.j.ii	LP1, LP3	**Context:** All mathematical context; no real life context used **Math focus:** Proofs with construction
2.15	K	152.i.iii	LP1, LP3, LP4, LP5	**Context:** All mathematical context; no real life context used **Math focus:** Students apply the triangle congruence theorems to parallelograms in order to generate and prove congruence theorems for parallelograms
Modeling Task- How Much Water?	M Introduce the idea of congruence when students are brainstorming ideas of different types of containers. Have students ponder the concept of scaling and how that relates to congruence.	152.d.i 152.d.ii	LP4, LP5, LP6	**Context:** Things we use that consume water **Math focus:** Volume, dilations, defining appropriate quantities for descriptive modeling

M153: Reasoning and Proof through Similarity

Illustrative Mathematics–Geometry Unit 3: Similarity

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- Consider giving access to dynamic geometry software such as GeoGebra or Desmos-Geometry throughout, so that students develop proficiency in using **technology as a tool for problem-solving and understanding (LP 7)**.
- The use of Math Language Routine 1: Stronger and Clearer Each Time may be useful in providing students opportunities to revisit their initial conjectures and make them stronger. By providing students opportunities to share their conjectures with each other and with the class, students can then go back and revise their own conjectures. There are opportunities for this MLR in Lessons 3, 5, 8, 9, and 14. This is necessary to fully meet content and practice expectations 153.d, 153.e, 153.f, and 153.g.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
153.a: Given a geometric figure and a dilation, draw the transformed figure using tools such as graph paper, tracing paper, or geometry software.	i. create a dilated figure using graph paper, tracing paper, or geometry software.
	ii. explain how you dilated a figure.
153.b: Specify a sequence of transformations that will carry a given figure onto a similar figure.	i. describe a sequence of transformations that would carry a figure onto a similar figure.
153.c: Verify experimentally the properties of dilations given by a center and a scale factor.	i. create a dilated figure using graph paper, tracing paper, or geometry software and describe the effect of dilations on line segments.
	ii. create a dilated figure using graph paper, tracing paper, or geometry software and describe the effect of dilations on angles.
153.d: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar.	i. write an argument or explanation for why two figures are similar, using similarity transformations in your reasoning.
	ii. write an argument or explanation for why two figures are not similar, using similarity transformations in your reasoning.
	iii. use the definition of dilation within your argument.
153.e: Using similarity transformations, explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	i. demonstrate, using similarity transformations an example, that similar figures have corresponding angles that are equal in measure.
	ii. explain why, in general, similar figures have corresponding angles that are equal in measure, using similarity transformations in your reasoning.
	iii. demonstrate, using an example, that similar figures have corresponding sides that are proportional.

Content and Practice Expectations	Indicators Choose an artifact where you…
	iv. explain why, in general, similar figures have corresponding sides that are proportional, using similarity transformations in your reasoning.
153.f: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	i. demonstrate, using an example, that if two figures have two corresponding angles that are equal in measure, then they are similar.
	ii. explain why, in general, two figures that have two corresponding angles that are equal in measure are similar, using similarity transformations in your reasoning.
153.g: Use similarity to prove theorems about triangles.	i. read a proof of a statement about a triangle or triangles in general and summarize or explain it in your own words.
	ii. write a proof of a statement about a triangle or triangles in general, using similarity transformations or similarity criteria in your reasoning.
153.h: Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	i. find an unknown length measurement in a triangle and explain how you determined your answer, using similarity transformations or similarity criteria in your reasoning.
	ii. find an unknown angle measurement in a triangle and explain how you determined your answer, using similarity transformations or similarity criteria in your reasoning.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	MT
153.a.i	X		X	X													
153.a.ii	X		X	X													

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	MT
153.b.i						X											
153.c.i			X	X			X										
153.c.ii			X	X			X										
153.d.i						X	X	X					X	X			
153.d.ii					X	X	X	X						X			
153.d.iii					X		X	X									
153.e.i				X	X	X	X						X				
153.e.ii				X	X	X	X						X				
153.e.iii			X	X	X		X						X				
153.e.iv			X		X		X						X				
153.f.i									X		X					X	
153.f.ii									X		X					X	
153.g.i								X						X			
153.g.ii					X			X	X		X		X	X			
153.h.i					X						X		X		X	X	X
153.h.ii									X							X	

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.1	K	153.a.i 153.a.ii	LP2, LP7	**Context:** Builds on student understanding of images, using common knowledge of scaled and distorted images **Math focus:** Scale drawings, dilations
3.2	D Listed as an optional lesson by authors. This lesson serves both as practice with proportional relationships and extension into an interesting context. Students also have the opportunity to work with large numbers and consider what they mean—how vast is space, really?			**Context:** Scale drawing of the planets **Math focus:** Creating and interpreting scaled drawings of figures
3.3	K Consider asking students to describe the effect of the dilations on the angles within this lesson to address CPE 153.c.ii.	153.a.i 153.a.ii 153.c.i 153.c.ii 153.e.iii 153.e.iv	LP2, LP4, LP7	**Context:** All mathematical context; no real life context used **Math focus:** Creating dilations given a scale factor and looking at the effect on side lengths

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.4	K	153.a.i 153.a.ii 153.c.i 153.c.ii 153.e.i 153.e.ii 153.e.iii	LP7	**Context:** All mathematical context; no real life context used **Math focus:** Verifying experimentally and asserting that dilations take angles to congruent angles; using this assertion to create a convincing argument that if dilations take angles to congruent angles, they must take lines to parallel lines
3.5	K Consider asking students to describe the relationship between corresponding angles within this lesson to fully address CPE 153.e.	153.d.ii 153.d.iii 153.e.i 153.e.ii 153.e.iii 153.e.iv 153.g.ii 153.h.i	LP1, LP3	**Context:** All mathematical context; no real life context used **Math focus:** Using the definition of dilation to prove that the triangles formed by connecting midpoints of two sides are dilations of the original triangle
3.6	K	153.b.i 153.d.i 153.d.ii 153.e.i 153.e.ii	LP3, LP4	**Context:** All mathematical context; no real life context used **Math focus:** Connecting similarity with transformations
3.7	K	153.c.i 153.c.ii 153.d.i 153.d.ii 153.d.iii	LP1, LP3	**Context:** All mathematical context; no real life context used **Math focus:** Using rigid transformations and dilations to reason about and make

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		153.e.i 153.e.ii 153.e.iii 153.e.iv		generalizations about similarity of triangles and other types of figures
3.8	K	153.d.i 153.d.ii 153.d.iii 153.g.i 153.g.ii	LP2, LP3	**Context:** All mathematical context; no real life context used **Math focus:** Proving figures to be similar or not similar
3.9	K	153.f.i 153.f.ii 153.g.ii 153.h.ii	LP3, LP5	**Context:** All mathematical context; no real life context used **Math focus:** Proving triangles are similar through the AA criterion
3.10	D Listed as an optional lesson by authors. No connection to CPEs.			**Context:** All mathematical context; no real life context used **Math focus:** Connection between SSS triangle congruence theorem and SSS similarity theorem, as well as connection between SAS triangle congruence theorem and SAS triangle similarity theorem

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.11	K	153.f.i 153.f.ii 153.g.ii 153.h.i	LP2, LP3, LP7	**Context:** All mathematical context; no real life context used **Math focus:** Parallel segments in triangles, finding the missing side lengths
3.12	D Listed as an optional lesson by authors. If choose to use, consider Amplifying Activity 12.3 to give students opportunity with 153.h.i. Lesson previews work with Pythagorean theorem and practice finding unknown values in proportional relationships. Consider Amplifying Activity 12.3.			**Context:** Garden spaces and quilts **Math focus:** Calculating unknown values in proportional relationships and finding scale factors
3.13	K	153.d.i 153.e.i 153.e.ii 153.e.iii 153.e.iv 153.g.ii 153.h.i	LP1, LP2, LP3, LP5	**Context:** All mathematical context; no real life context used **Math focus:** Using Pythagorean theorem and similarity to solve missing side lengths

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.14	K	153.d.i 153.d.ii 153.g.i 153.g.ii	LP3, LP5	**Context:** All mathematical context; no real life context used **Math focus:** Proving the Pythagorean theorem using similar triangles
3.15	M Prioritize 15.1 and 15.2 to focus on finding unknown lengths of a triangle and explaining the use of similarity criteria.	153.h.i	LP3, LP4	**Context:** All mathematical context; no real life context used **Math focus:** Finding unknown lengths of similar right triangles
3.16	K Consider asking students to describe the relationship between corresponding angles within this lesson to fully address CPE 153.h.	153.f.i 153.f.ii 153.h.i 153.h.ii	LP1, LP3, LP5, LP6	**Context:** Shooting pool, measuring tall objects with mirrors **Math focus:** Using similarity to solve problems

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task - Scaling a Playground	K	153.h.i	LP1, LP2, LP3, LP4, LP6	**Context:** Tracing the path of a cup rolling on a flat surface **Math focus:** Applying theorems about circles, using geometric shapes and their measures/properties to describe objects, using congruence and similarity criteria for triangles to solve problems

M154: Coordinate Geometry

Illustrative Mathematics–Geometry Unit 6: Coordinate Geometry

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum and opportunities to use the embedded math language routines to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense

of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
154.a: Translate between the geometric description and the equation for a conic section, specifically of a circle and parabola.	i. explain how the Pythagorean theorem can be used to derive the equation of a circle.
	ii. write the equation for a parabola using the focus and directrix.
154.b: Use coordinates to prove simple geometric theorems algebraically.	i. use the distance formula to find the distance between two points on a coordinate plane.
	ii. use coordinates along with the slope formula, distance formula, and/or midpoint formula to prove simple geometric theorems algebraically.
	iii. use coordinates to prove that a quadrilateral is, or is not, a specific quadrilateral.
154.c: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.	i. classify lines or segments as parallel, perpendicular, or neither.
	ii. write the equation of a line parallel or perpendicular to a given line and passing through a given point.
	iii. solve problems involving equations of parallel and perpendicular lines.
154.d: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	i. use the midpoint formula, the section formula, and the distance formula to find the partition point of a given directed line segment.

Content and Practice Expectations	Indicators Choose an artifact where you…
	ii. use dilations to find the coordinates of a point that divides a directed line segment in a given ratio.
154.e: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).	i. use coordinates to compute perimeters of polygons and the areas of triangles and quadrilaterals.
	ii. decompose complex figures whose vertices can be located on a coordinate plane into simple shapes to determine perimeter and/or area.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	MT
154.a.i				X	X	+												
154.a.ii							X	X										
154.b.i				X			X	X										
154.b.ii				X			X	X							X	X	X	
154.b.iii														X				
154.c.i										X	X	X					X	
154.c.ii										X	X	X	X				X	
154.c.iii										X	X	X	X				X	
154.d.i															X	X		

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	MT
154.d.ii															X			
154.e.i														X				X
154.e.ii																		X

Note: Columns in gray indicate a recommendation to delete the lesson

+ This lesson supports work at the level of the Content and Practice Expectation, yet not at the indicator level.

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.1	D No connection to CPEs, but may be helpful for establishing a foundational understanding of rigid transformations on the coordinate plane.			**Context:** Purely mathematical **Math focus:** Rigid transformations, congruence, finding length on the coordinate plane
6.2	D No connection to CPEs, but may be helpful for establishing a foundational understanding of transformations as functions on the coordinate plane.			**Context:** Purely mathematical **Math focus:** Rigid transformations and dilations on the coordinate plane
6.3	D No connection to CPEs, but may be helpful for building a lens to analyze transformations that produce congruent			**Context:** Purely mathematical **Math focus:** Writing rules for transformations on the coordinate plane; comparing figures as congruent or similar

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	and similar figures.			
6.4	K	154.a.i 154.b.i 154.b.ii	LP1, LP3	**Context:** Purely mathematical **Math focus:** Deriving the equation of a circle on coordinate plane; using the Pythagorean theorem to find the distance between two points
6.5	M Focus on Activity 5.3 to solidify an understanding of the equation of a circle. Other activities could be useful for solidifying an understanding of the connection between the distributive property and perfect square trinomials.	154.a.i	LP1	**Context:** Purely mathematical **Math focus:** Continued development of understanding for the equation for a circle by looking at perfect square trinomials
6.6	M Use work on Activity 6.2 and Activity 6.3 to connect the structure of the equation of a circle	154.a	LP3	**Context:** Purely mathematical **Math focus:** Finding the center and radius of a circle by completing the square

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	to the graph on the coordinate plane. This lesson will support work to CPE 154.a with that emphasis. No direct connection to indicators associated with 154.a.			
6.7	K	154.a.ii 154.b.i 154.b.ii	LP1, LP7	**Context:** Purely mathematical **Math focus:** Transition from circles to parabolas; understanding the focus and directrix; identifying an equation for a parabola
6.8	K	154.a.ii 154.b.i 154.b.ii	LP1, LP2, LP6, LP7	**Context:** Purely mathematical **Math focus:** Writing an equation to represent a parabola using distance between a point and a directrix
6.9	D No connection to CPEs, yet may be helpful to bolster an understanding of point-slope form for an			**Context:** Purely mathematical **Math focus:** Moving from circles/parabolas to lines; analyzing slope; writing equations in point-slope form

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	equation.			
6.10	K	154.c.i 154.c.ii 154.b.iii	LP1, LP3, LP7	**Context:** Purely mathematical **Math focus:** Proving parallel lines
6.11	K	154.c.i 154.c.ii 154.c.iii	LP1, LP3, LP7	**Context:** Purely mathematical **Math focus:** Using transformations to show perpendicular lines; working with perpendicular lines and equations
6.12	K	154.c.i 154.c.ii 154.c.iii	LP2, LP4	**Context:** Purely mathematical **Math focus:** Using an understanding of parallel and perpendicular lines to write equations; solving problems involving parallel and perpendicular lines
6.13	M Emphasize Activity 13.3 if needed to continue an understanding of parallel/perpendicular lines going through a certain point. If not needed, consider	154.c.ii 154.c.iii	LP3	**Context:** Purely mathematical **Math focus:** Writing equations for parallel/perpendicular lines that intersect a circle

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	deleting this lesson.			
6.14	K	154.b.iii 154.e.i	LP2, LP4	**Context:** Purely mathematical **Math focus:** Finding measurements of quadrilaterals on the coordinate plane; using slope to prove right angles are made by perpendicular lines
6.15	M Emphasize Activity 15.3 question 4 in order to focus on indicator 154.d.ii.	154.b.ii 154.d.i 154.d.ii	LP1, LP3, LP6	**Context:** Purely mathematical **Math focus:** Finding a partitioning point; creating lines/shapes with given ratios
6.16	K	154.b.ii 154.d.i	LP1, LP2, LP4	**Context:** Purely mathematical **Math focus:** Creating medians using midpoints; proving intersection at the centroid
6.17	K	154.b.ii 154.c.i 154.c.ii 154.c.iii	LP1, LP3	**Context:** Purely mathematical **Math focus:** Using midpoint/slope to understand altitudes, medians, and perpendicular bisectors of triangles on coordinate plane

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task- A New Town	M Have students do research similar to what urban and regional planners do to learn about zoning and building codes, environmental regulations, and other issues that need to be considered when planning for use of land. Consider having students use software like GeoGebra or Desmos- Geometry to design their town to engage with additional CPEs.	154.e.i 154.e.ii	LP1, LP2, LP3, LP4, LP6	**Context:** Students will design a town **Math focus:** Computing area/perimeter of polygons

M155: Right Triangle Trigonometry

Illustrative Mathematics– Geometry Unit 4: Right Triangle Trigonometry

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum and embedded instructional routines to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- Opportunities for students to **employ technology as a tool for problem-solving and understanding (LP7)** are available throughout the unit, particularly the use of calculators when using ratios and using trigonometric functions for solving missing side lengths and angles of right triangles.
- Consider adding in supplemental activities to support 155.c.ii with contextual practice problems from the curriculum.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students

engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
155.a: Use similarity to explain the meaning of trigonometric ratios.	i. measure side lengths and angles of multiple triangles and describe patterns you see.
	ii. explain why trigonometric ratios remain the same for different-sized right triangles.
155.b: Explain and use the relationship between the sine and cosine of complementary angles.	i. illustrate the relationship between the sine and cosine of complementary angles.
	ii. use the relationship between the sine and cosine of complementary angles to solve problems.
155.c: Use trigonometric ratios to solve right triangles in applied problems.	i. use trigonometric ratios to find unknown side lengths in problems.
	ii. use trigonometric ratios to find unknown angles in problems.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	MT
155.a.i	X	X	X	X	X							X
155.a.ii				X	X							
155.b.i								X				
155.b.ii								X		X		
155.c.i					X	X	X		X	X	X	X
155.c.ii									X	X		X

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
4.1	K	155.a.i	LP5, LP6	**Context:** Ramps for wheelchairs, ramps for schools **Math focus:** Angles and ratios of right triangles
4.2	M Combine Lesson 4.2 and Lesson 4.3.	155.a.i	LP3	**Context:** Purely mathematical **Math focus:** 45-45-right triangle as half a square
4.3	M Combine Lesson 4.2 and Lesson 4.3.	155.a.i	LP3	**Context:** Purely mathematical **Math focus:** 30-60-right triangle as half an equilateral triangle
4.4	K	155.a.i 155.a.ii	LP1, LP3, LP5	**Context:** Purely mathematical **Math focus:** Connect angle measures with ratios of side lengths of right triangles
4.5	K	155.a.i 155.a.ii 155.c.i	LP3	**Context:** Rocket launch (warm-up) **Math focus:** Using ratios in right

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
				triangles to solve missing side lengths and introduction of Pythagorean triples
4.6	K	155.c.i	LP2, LP3	**Context:** Purely mathematical **Math focus:** Formal introduction of trigonometric ratios by name and continued solving triangles
4.7	K	155.c.i	LP2, LP3, LP5, LP6	**Context:** Height of Burj Khalifa and City Hall in Philadelphia **Math focus:** Using trigonometric ratios to solve missing side lengths in context
4.8	K	155.b.i 155.b.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Relationship between sine and cosine of complementary angles
4.9	K	155.c.i 155.c.ii	LP3, LP5, LP6	**Context:** Safe angles of a leaning ladder **Math focus:** Using trigonometric

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
				ratios to solve unknown angles of right angles
4.10	K	155.b.ii 155.c.i 155.c.ii	LP1, LP3	**Context:** Airplane's angle of descent **Math focus:** Solving problems using trigonometry
4.11	K	155.c.i	LP1, LP3, LP5	**Context:** Purely mathematical **Math focus:** Generalizing a formula for the perimeter of a polygon inscribed in a circle
Modeling Task- So Many Flags	M Consider having students construct a replica of the flag they select on GeoGebra or Desmos-Geometry.	155.a.i 155.c.i 155.c.ii	LP1, LP3, LP4, LP5, LP6, LP7	**Context:** Flag of Nepal, parade with flags **Math focus:** Constructions, using Pythagorean theorem and right triangle trigonometry to solve for side lengths and angles of a flag of their choice

M201: Function Concepts

Illustrative Mathematics–Algebra 2 Unit 1: Sequences and Functions and Algebra 2 Unit 5: Transformations of Functions and Algebra 1 Unit 7: Quadratic Equations

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
201.a: Understand the concept of a function and use function notation.	i. demonstrate understanding of the following language of functions: If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.
	ii. recognize that sequences are functions, sometimes defined recursively, and their domain is a subset of the integers.
201.b: **Graph** functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	i. graph linear and quadratic functions and show intercepts, maxima, and minima.
	ii. graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
	iii. graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
	iv. graph exponential functions showing intercepts and end behavior.
201.c: **Write** a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	i. rewrite expressions, using processes such as factoring and completing the square in a quadratic function, to show zeros, extreme values, and symmetry of graphs.
	ii. use the properties of exponents to interpret expressions for exponential functions.
201.d: **Build** new functions from existing functions.	i. identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
201.e: **Compare** properties of two functions each represented in a different	i. compare properties of two functions that highlight two quantities or features of interest.

Content and Practice Expectations	Indicators Choose an artifact where you…
way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
201.f: **Build** a function that models a relationship between two quantities.	i. identify and/or define the variables of interest in a given situation or data set that demonstrate a functional relationship.
	ii. build a function that models the relationship between two quantities in a situation.

Unit Overview—Algebra 2 Unit 1: Sequences and Functions

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	MT1
201.a.i			X		X	X	X	X	X	X	X	
201.a.ii	X	X	X		X	X	X	X	X	X	X	X
201.b.i			X				X					
201.b.ii												
201.b.iii												
201.b.iv		X				X	X					

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	MT1
201.c.i												
201.c.ii								X	X			
201.d.i												
201.e.i						X		X		X		
201.f.i								X	X	X	X	X
201.f.ii					X	X	X	X	X	X	X	X

Note: Columns in gray indicate a recommendation to delete the lesson

Unit Overview—Algebra 2 Unit 5: Transformations of Functions and Algebra 1 Unit 7: Quadratic Equations

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	MT2	L12	L13	L14	L15	L16	L17	MT3
201.a.i	X	X	X	X		X	X	X	X	X	X		X	X				X	X
201.a.ii																			
201.b.i	X							X		X	X	X	X			X		X	X
201.b.ii		X	X						X	X	X	X							
201.b.iii	X	X		X		X		X		X									
201.b.iv	X		X				X			X	X								
201.c.i														X	X	X	X	X	X
201.c.ii																			
201.d.i	X	X	X	X	X	X	X	X	X		X	X							X
201.e.i			X	X	X				X										
201.f.i												X							
201.f.ii		X	X				X			X	X	X							X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority—Algebra 2 Unit 1: Sequences and Functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
1.1 Unit 1	K	201.a.ii	LP1, LP2, LP3, LP6	**Context:** The Tower of Hanoi **Math focus:** Giving examples of sequences (described recursively)
1.2	K	201.a.ii 201.b.iv	LP2, LP3, LP6	**Context:** Paper slicing **Math focus:** Finding missing terms in a geometric sequence
1.3	K	201.a.i 201.a.ii 201.b.i	LP2, LP3	**Context:** Purely mathematical **Math focus:** Explaining what it means for a sequence to be arithmetic or geometric
1.4	D Lesson listed as optional by authors. The focus is on learning to use spreadsheets and graphing technology without context. Recommend including use of tech tools, like graphing calculators and			**Context:** Purely mathematical—use of spreadsheets **Math focus:** Using technology to graph a sequence and create many terms of a sequence

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	spreadsheets, within other lessons so that LP7 is integrated into learning experiences.			
1.5	K	201.a.i 201.a.ii 201.f.ii	LP2, LP3	**Context:** Visual pattern of dots **Math focus:** Defining arithmetic and geometric sequences recursively using function notation
1.6	K Lesson is listed as optional by the authors. Yet, this lesson provides more practice writing recursive definitions.	201.a.i 201.a.ii 201.b.iv 201.e.i 201.f.ii	LP2, LP3	**Context:** Purely mathematical—visual patterns **Math focus:** Representing sequences in different ways
1.7	K	201.a.i 201.a.ii 201.b.i 201.b.iv 201.f.ii	LP2, LP3, LP5	**Context:** Purely mathematical **Math focus:** Asking questions to get information to represent a sequence

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
1.8	K	201.a.i 201.a.ii 201.c.ii 201.e.i 201.f.i 201.f.ii	LP2, LP3, LP5	**Context:** Paper slicing; a Sierpinski triangle **Math focus:** Explaining why different equations can represent the same sequence
1.9	K	201.a.i 201.a.ii 201.c.ii 201.f.i 201.f.ii	LP2, LP3, LP6	**Context:** Taking cake; Fibonacci squares **Math focus:** Representing situations with sequences
1.10	K	201.a.i 201.a.ii 201.e.i 201.f.i 201.f.ii	LP2, LP3, LP6, LP7	**Context:** Population; square patterns **Math focus:** Defining sequences using equations
1.11	K	201.a.i 201.a.ii 201.f.i 201.f.ii	LP2, LP3, LP6	**Context:** Paper trail; Koch snowflake **Math focus:** Determining the sum of a sequence representing a situation

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task 1- Viral Marketing	Three versions are given, with a varied amount of information given to students.	201.a.ii 201.f.i 201.f.ii	LP1, LP3, LP4, LP6, LP7	**Context:** Marketing campaign **Math focus:** Modeling different scenarios with exponential functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.1	K	201.a.i 201.b.i 201.b.iii 201.b.iv 201.d.i	LP2, LP3, LP6	**Context:** Soda water temperature; describing and sketching transformed graphs **Math focus:** Describing how a graph is transformed

Guidance for Lesson Priority—Algebra 2 Unit 5: Transformations of Functions and Algebra 1 Unit 7: Quadratic Equations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.2	K	201.a.i 201.b.ii 201.b.iii 201.d.i 201.f.ii	LP3, LP6	**Context:** A bakery kitchen's temperature **Math focus:** Using function notation to represent a vertical or horizontal translation
5.3	K	201.a.i 201.b.ii 201.b.iv 201.d.i 201.e.i 201.f.ii	LP3, LP6	**Context:** Bakery kitchen; thawing meat **Math focus:** Writing equations to represent vertical and horizontal translations
5.4	K	201.a.i 201.b.iii 201.d.i 201.e.i	LP2, LP3	**Context:** Purely mathematical **Math focus:** Reflecting a graph across the x- and y-axis
5.5	K	201.d.i 201.e.i	LP1, LP2, LP3	**Context:** Ferris wheel **Math focus:** Identifying even and odd functions by their graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.6	K	201.a.i 201.b.iii 201.d.i	LP2, LP3	**Context:** Purely mathematical **Math focus:** Completing even and odd functions given half the graph; identifying even and odd functions by their equations
5.7	K	201.a.i 201.b.iv 201.d.i 201.f.ii	LP3, LP6, LP7	**Context:** Temperature of soda water left out **Math focus:** Writing an equation from a description of how a graph is transformed
5.8	K	201.a.i 201.b.i 201.b.iii 201.d.i	LP3, LP6, LP7	**Context:** Hulme Arch Bridge; feeding the dog **Math focus:** Calculating the scale factor needed to transform the output of a function to model data
5.9	K	201.a.i 201.b.ii 201.d.i 201.e.i	LP2, LP3, LP6	**Context:** Distance walked and Ferris wheel **Math focus:** Describing the effect of a scale factor on the input of a function; understanding the differences between scaling the outputs and scaling the inputs of a function

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.10	K Students are combining functions given in graphical form to deepen conceptual underpinnings of CPE.201.b.	201.a.i 201.b.i 201.b.ii 201.b.iii 201.b.iv 201.f.ii	LP2, LP3, LP6	**Context:** Book sales **Math focus:** Combining two functions in different ways
5.11	K	201.a.i 201.b.i 201.b.ii 201.b.iv 201.d.i 201.f.ii	LP1, LP2, LP3, LP6, LP7	**Context:** Temperature of bottled water **Math focus:** Transforming a function so its graph models a data set
Modeling Task 2- Swing Time	Two versions are given, with more information given in the second.	201.b.i 201.b.ii 201.d.i 201.f.i 201.f.ii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Period of a pendulum **Math focus:** Building a best-fit function using transformations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
7.2 Algebra 1 Unit 7	M Replace Activity 2.1 with warm-up from A1 Unit 7 Lesson 1: What goes up must come down to launch this lesson.	201.a.i 201.b.i	LP3, LP6, LP7	**Context:** Potato launch; ticket sales revenues **Math focus:** Connect factored and standard form of quadratic functions and see solution in graph of the function
7.3 and 7.4	M Merge these two lessons as recommended below: Activity 3.1, Activity 3.2, Activity 3.2: Are you ready for more?, and Activity 4.3.	201.a.i 201.c.i	LP1, LP3, LP6, LP7	**Context:** Projectile motion **Math focus:** Zero product property
7.5	M Foundation for CPE 201.c.i. Amplify Activities 5.2 and 5.4.	201.c.i	LP1, LP3	**Context:** Purely mathematical **Math focus:** Graphing and rewriting quadratic equations in the form of expression = 0
7.6, 7.7, and 7.9	M Merge these lessons as recommended below: Activity 6.2, Activity 7.2, Activity 7.3, and Activity	201.b.i 201.c.i	LP1, LP3, LP7	**Context:** Purely mathematical **Math focus:** Rewriting quadratic expressions in factored form

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	9.3.			
7.11, 7.12, and 7.14	M Merge these lessons as recommended below: Activity 11.2, Activity 12. 2, Activity 12.3, and Activity 14.3.	201.c.i	LP1, LP3, LP7	**Context:** Purely mathematical **Math focus:** Perfect squares and completing the square
7.23	K	201.b.i 201.c.i	LP1, LP3, LP7	**Context:** Ticket sales **Math focus:** Determining maximum and minimum values of a quadratic function
Modeling Task 3- Planning a Concert	K	201.a.i 201.b.i 201.c.i 201.d.i 201.f.ii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Optimizing profit for a concert **Math focus:** Using a quadratic function to determine optimal price for a concert

M202: Rational Exponents and Complex Numbers

Illustrative Mathematics–Algebra 2 Unit 3-Complex Numbers and Rational Exponents

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense

of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
202.a: Reason about and extend the properties of exponents to rational exponents.	i. explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, and how this definition leads to notation for radicals in terms of rational exponents.
	ii. rewrite expressions involving radicals and rational exponents using the properties of exponents.
202.b: Reason about and perform operations with complex numbers.	i. demonstrate understanding that there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
	ii. use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
202.c: Analyze and use complex numbers in polynomial equations.	i. solve quadratic equations with real coefficients that have complex solutions.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	MT
202a.i			X	X	X															
202.a.ii			X	X	X															

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	MT
202.b.i							X			X	X		X	X						
202.b.ii												X	X	X						
202.c.i																X	X	X		

Note: Columns in gray indicate a recommendation to delete the lesson.

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.1	D This lesson is listed as optional by authors. This lesson reviews exponent rules that were developed in grade 8 where the exponents were integers. Use this lesson as needed.			**Context:** Purely mathematical **Math focus:** Properties of exponents, where exponents are integers; and evaluating expressions with integer exponents
3.2	D This lesson is listed as optional by authors. This lesson reviews square and cube roots. Use this			**Context:** Purely mathematical; use of area and volume as context **Math focus:** Calculating square and cube roots

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	lesson as needed.			
3.3	K	202.a.i 202.a.ii	LP1, LP2, LP3, LP5, LP7	**Context:** Purely mathematical **Math Focus:** Writing square and cube roots as rational exponents
3.4	M Prioritize Activity 4.3. One way to do this is to embed some of the questions from Activity 4.2 within the discussion of Activity 4.3.	202.a.i 202.a.ii	LP1, LP3, LP7	**Context:** Purely mathematical **Math Focus:** Using graphs to estimate the value of expressions involving positive rational exponents
3.5	K	202.a.i 202.a.ii	LP2, LP3, LP7	**Context:** Purely mathematical **Math Focus:** Using graphs to estimate the value of expressions involving negative rational exponents
3.6	D No connection to CPEs.			**Contexts:** Purely mathematical **Math Focus:** Understanding that the square root symbol means the positive square root; seeing solutions to $x^2 = a$ and $y = \sqrt{x}$ in graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.7	M Prioritize Activities 3.7.2 and 3.7.3.	202.b.i	LP1, LP2, LP3	**Context:** Purely mathematical **Math Focus:** Reason about solutions to equations involving squares and square roots
3.8	D No connection to CPEs.			**Context:** Purely mathematical **Math Focus:** Solving cube root and cubic equations algebraically
3.9	D This lesson is listed as optional by authors. It is an opportunity for extra practice in solving a variety of radical equations.			**Context:** Purely mathematical **Math Focus:** Solving equations with radicals in them
3.10	K	202.b.i	LP1, LP3, LP5	**Context:** Purely mathematical **Math Focus:** Representing imaginary numbers visually with the imaginary number line

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.11	K	202.b.i	LP1, LP3, LP5, LP6	**Context:** Purely mathematical **Math Focus:** Using i to solve quadratic equations and representing solutions to equations using i and the complex plane
3.12	M Prioritize Activity 12.2. Activity 12.3 sets the stage for the next lesson—consider using it at some point to introduce Lesson 13.	202.b.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math Focus:** Using the complex plane to add complex numbers; calculating powers of imaginary numbers
3.13	K	202.b.i 202.b.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math Focus:** Multiplying complex numbers
3.14	K	202.b.i 202.b.ii	LP3	**Context:** Purely mathematical **Math Focus:** Practicing adding, subtracting, and multiplying complex numbers

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.15	D Recommend to use if seeing that extra time on this will support students. Good lesson to engage students in reasoning about complex numbers as a way to synthesize learning.			**Context:** Purely mathematical **Math Focus:** Finding real and imaginary parts of complex numbers if given enough information about the numbers and their product
3.16	K	202.c.i	LP1, LP3, LP4	**Context:** Purely mathematical **Math Focus:** Solving quadratic equations by completing the square or by using the quadratic formula
3.17	K Please note Activity 17.4 is listed as optional in this lesson. This allows you to prioritize 17.2 and 17.3.	202.c.i	LP1, LP3	**Context:** Purely mathematical **Math Focus:** Solving quadratic equations by completing the square

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.18	K	202.c	LP1, LP3	**Context:** Purely mathematical **Math Focus:** Finding complex solutions to quadratic equations by using the quadratic formula
3.19	D Listed as optional by authors.			**Context:** Using the value under the radical $b^2 - 4ac$ to determine the number of real solutions **Math Focus:** Determining how many real solutions a quadratic equation has

M203: Polynomial and Rational Expressions, Functions, and Equations

Illustrative Mathematics–Algebra 2 Unit 2- Polynomial and Rational Functions

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense

of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
203.a: Interpret the structure of polynomial and rational expressions.	i. interpret how parts of an expression, such as terms, factors, and coefficients relate to a real-world situation.
	ii. interpret complicated expressions by viewing one or more of their parts as a single entity.
	iii. use the structure of an expression to identify ways to rewrite it.
203.b: Reason about and perform operations on polynomials.	i. demonstrate understanding that polynomials are closed under the operations addition, subtraction, and multiplication.
	ii. add, subtract, and multiply polynomials.
203.c: Understand the relationship between the zeros and factors of polynomials.	i. identify zeros of polynomials when suitable factorizations are available.
	ii. use the zeros to construct a rough graph of a function defined by a polynomial.
203.d: Use polynomial identities to solve problems.	i. prove polynomial identities and use them to describe a numerical relationship.
203.e: Rewrite rational expressions.	i. rewrite simple rational expressions in different forms.
203.f: Interpret polynomial and rational functions that arise in applications in terms of the context.	i. interpret key features of graphs and tables in terms of the quantities for a function that models a relationship between two quantities.

Content and Practice Expectations	Indicators Choose an artifact where you…
	ii. sketch graphs showing key features given a verbal description of the relationship between quantities.
	iii. calculate and interpret the average rate of change of a polynomial or rational function over a specified interval.
	iv. relate the domain of a function to its graph, and where applicable, to the quantitative relationship it describes.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	L22	L23	L24	L25	L26	MT
203.a.i	X	X				X										X	X	X	X	X	X				X	X	X
203.a.ii	X	X	X			X		X	X	X				X		X	X	X	X	X	X	X			X	X	X
203.a.iii	X	X	X			X			X			X	X	X		X		X	X	X	X	X	X	X	X	X	
203.b.i				X		X						X	X	X									X		X		
203.b.ii				X		X						X	X	X									X		X		
203.c.i	X		X		X	X	X		X	X		X		X													
203.cii			X		X	X	X		X	X		X		X													X
203.d.i																							X	X	X	X	
203.e.i																X		X	X	X	X	X		X		X	

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	L22	L23	L24	L25	L26	MT
203.f.i	X	X	X			X		X		X				X		X	X	X	X	X	X				X		X
203.f.ii	X	X							X					X			X	X	X		X						X
203.f.iii		X																									X
203.f.iv	X	X				X										X		X									X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.1	K Attend to 203.a.iii, look for a variety of equivalent expressions in Activity 1.3.	203.a.i 203.a.ii 203.a.iii 203.c.i 203.f.i 203.f.ii 203.f.iv	LP1, LP2, LP3, LP6, LP7	**Context:** Building a box **Math focus:** Finding the volume; creating a polynomial
2.2	K	203.a.i 203.a.ii 203.a.iii 203.f.i 203.f.ii 203.f.iii 203.f.iv	LP1, LP3, LP6, LP7	**Context:** Investing money **Math focus:** Using a polynomial to model a situation
2.3	K	203.a.ii 203.a.iii 203.c.i 203.c.ii 203.f.i	LP1, LP2, LP3, LP7	**Context:** Purely mathematical **Math focus:** Identifying characteristics of polynomial graphs and expressions
2.4	K	203.b.i 203.b.ii	LP2, LP3	**Context:** Purely mathematical **Math focus:** Adding, subtracting,

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
				multiplying polynomials
2.5	K	203.c.i 203.c.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Finding the zeros from the factored form
2.6	K Consider asking students to describe the domain of the function in focus on Activity 6.2 to get to full extent of 203.f.iv.	203.a.i 203.a.ii 203.a.iii 203.b.i 203.b.ii 203.c.i 203.c.ii 203.f.i 203.f.iv	LP1, LP2, LP3, LP6, LP7	**Context:** Revisits box from Lesson 1 **Math focus:** Identifying features of a polynomial using standard and factored forms
2.7	K	203.c.i 203.c.ii	LP1, LP3, LP7	**Context:** Choosing proper window settings **Math focus:** Writing polynomials given the zeros
2.8	K	203.a.ii 203.f.i	LP1, LP3, LP7	**Context:** Purely mathematical **Math focus:** Understanding why the leading term determines end behavior

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.9	K	203.a.ii 203.a.iii 203.c.i 203.c.ii 203.f.ii	LP1, LP2, LP3, LP7	**Context:** Purely mathematical **Math focus:** Using the leading term to determine end behavior
2.10	K	203.a.ii 203.c.i 203.c.ii 203.f.i	LP1, LP2, LP3, LP7	**Context:** Purely mathematical **Math focus:** Using zeros and multiplicities to sketch a graph of a polynomial
2.11	D Not aligned to expectations of this badge and not needed for coherence.			**Context:** Purely mathematical **Math focus:** Finding where two polynomials intersect
2.12	K Division of polynomials is the context for engaging in 203.b. Amplify the underlying reasoning.	203.a.iii 203.b.i 203.b.ii 203.c.i 203.c.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Dividing polynomials using diagrams

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.13	K Division of polynomials is the context for engaging in 203.b. Amplify the underlying reasoning.	203.a.iii 203.b.i 203.b.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Dividing polynomials using long division
2.14	K	203.a.ii 203.a.iii 203.b.i 203.b.ii 203.c.i 203.c.ii 203.f.i 203.f.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Rewriting a polynomial in factored form starting from a known factor
2.15	D Not aligned to expectations of this badge and not needed for coherence.			**Context:** Purely mathematical **Math focus:** Remainder theorem
2.16	K	203.a.i 203.a.ii 203.a.iii 203.e.i 203.f.i	LP1, LP2, LP3, LP7	**Context:** Minimizing surface area of a cylinder **Math focus:** Writing rational functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		203.f.iv		
2.17	K	203.a.i 203.a.ii 203.f.i 203.f.ii	LP1, LP2, LP3, LP6, LP7	**Context:** Distance, rate, and time relationship for a bike ride **Math focus:** Identifying vertical asymptotes
2.18	K	203.a.i 203.a.ii 203.a.iii 203.e.i 203.f.i 203.f.ii 203.f.iv	LP1, LP2, LP3, LP6	**Context:** Cost of publishing a book **Math focus:** Identifying horizontal asymptotes
2.19	K	203.a.i 203.a.ii 203.a.iii 203.e.i 203.f.i 203.f.ii	LP1, LP2, LP3, LP6	**Context:** Combined fuel efficiency **Math focus:** Finding end behavior of rational functions by rewriting as f(x) = q(x) + r(x)/b(x)
2.20	K	203.a.i 203.a.ii 203.a.iii 203.e.i	LP1, LP2, LP3, LP6, LP7	**Context:** Price of t-shirts; batting averages **Math focus:** Writing rational expressions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		203.f.i		that represent averages
2.21	K	203.a.i 203.a.ii 203.a.iii 203.e.i 203.f.i 203.f.ii	LP1, LP2, LP3, LP6, LP7	**Context:** Boat ride on river; parallel circuits **Math focus:** Writing and solving equations with simple rational expressions on both sides
2.22	K	203.a.ii 203.a.iii 203.e.i	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Checking for extraneous solutions to rational equations
2.23	K	203.a.iii 203.b.i 203.b.ii 203.d.i	LP1, LP2, LP3, LP5	**Context:** Purely mathematical **Math focus:** Understanding what an identity is in mathematics; investigating the difference of squares and x^n-1.
2.24	K	203.a.iii 203.d.i 203.e.i	LP1, LP2, LP3, LP5	**Context:** Sides of a right triangle; Egyptian fractions **Math focus:** Justifying why identities are true

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.25	K	203.a.i 203.a.ii 203.a.iii 203.b.i 203.b.ii 203.d.i 203.f.i	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Koch snowflake; antibiotics **Math focus:** Understanding why the geometric sum formula is true
2.26	K	203.a.i 203.a.ii 203.a.iii 203.d.i 203.e.i	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** House sales; savings account **Math focus:** Using the geometric sum formula to solve problems
Modeling Task-The Paths of the Planets	Two versions are given—the first three questions are the same, but the fourth gives data in one version and has students research to find the data in the second version. To increase alignment to CPEs listed, ask students to interpret key features of the graph and describe the domain.	203.a.i 203.a.ii 203.c.ii 203.f.i 203.f.ii 203.f.iii 203.f.iv	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Kepler's relationship between planets' orbital periods and their distance from the sun **Math focus:** Modeling with polynomials

M204: Exponential and Logarithmic Functions and Equations

Illustrative Mathematics–Algebra 2 Unit 4: Exponential Functions and Equations

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum, such as math language routines, to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting "doing mathematics," but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
204.a: Understand the relationship between exponential and logarithmic functions.	i. use reasoning about exponents to explain the value of a logarithmic expression.
	ii. write an exponential expression to verify the value of a logarithmic expression.
204.b: Graph exponential and logarithmic functions.	i. graph an exponential function over a domain that includes non-integers.
	ii. graph a logarithmic function.
	iii. explain the effect of changing the parameters in an exponential or logarithmic function on its graph.
204.c: Interpret logarithmic and exponential functions that arise in applications in terms of the context.	i. interpret key features of the graph of an exponential function in terms of the context being modeled.
	ii. interpret key features of the graph of a logarithmic function in terms of the context being modeled.
	iii. explain the choice of domain for a logarithmic or exponential function in terms of the context being modeled.
	iv. calculate and explain the meaning of the average rate of change for an exponential or logarithmic function in terms of the context being modeled.
204.d: Use logarithms to analyze exponential models.	i. create an exponential model for a context.
	ii. use a logarithm to solve for an unknown value of an exponential model.
	iii. interpret values for exponential models in context.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	MT
204.a.i									X	X	X						X		
204.a.ii									X	X	X			X			X		
204.b.i			X	X	X	X	X	X											X
204.b.ii															X		X	X	X
204.b.iii					X	X							X			X	X		
204.c.i			X	X	X	X						X	X		X	X			X
204.c.ii															X		X	X	
204.c.iii			X	X	X												X	X	X
204.c.iv			X	X	X	X	X	X											X
204.d.i	X	X	X	X	X	X	X	X				X	X						
204.d.ii										X	X			X	X				
204.d.iii		X	X	X		X	X						X		X	X			

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
4.1	K The content lays a foundation for identified CPEs.	204.d.i	LP1, LP2, LP3, LP6	**Context:** Shrinking a passport photo; pond in a park **Math focus:** Calculating values that are changing exponentially
4.2	K Listed as an optional lesson by authors, but a good opportunity to address CPEs listed. Consider asking students for the exponential model in Activity 2.3 for stronger connection to CPE 204.d.i.	204.d.i 204.d.iii	LP1, LP2, LP3, LP6	**Context:** Paying Grandma back; climbing cost of college tuition; decreasing value of two vans **Math focus:** Seeing that exponential functions change by equal factors over equal intervals
4.3	M Leverage Activity 3.2 to fully address CPE 204.b and CPE 204.c by explicitly asking students about reasonable domains	204.b.i 204.c.i 204.c.iii 204.c.iv 204.d.i 204.d.iii	LP1, LP2, LP3, LP6, LP7	**Context:** Population growth in Florida; disappearing medicine **Math focus:** Determining the value of exponential functions at non-whole number inputs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	for the functions used and connections between the function, graph, and context.			
4.4	K	204.b.i 204.c.i 204.c.iii 204.c.iv 204.d.i 204.d.iii	LP1, LP3, LP6, LP7	**Context:** Population growth in Nigeria; half-life of caffeine **Math focus:** Calculating a growth or decay factor of an exponential function for different input values
4.5	M Leverage activities to address CPE 204.b and CPE 204.c by explicitly asking students about reasonable domains for the functions used and connections between the function, graph, and context.	204.b.i 204.b.iii 204.c.i 204.c.iii 204.c.iv 204.d.i	LP1, LP2, LP3, LP7	**Context:** Machine depreciation; fever medicine concentration in body **Math focus:** Explaining why exponential functions change by the same factor over equal intervals, even when the intervals are not whole numbers
4.6	K	204.b.i 204.b.iii 204.c.i 204.c.iv	LP1, LP2, LP3	**Context:** Bacteria growth expressions **Math focus:** Writing equations for exponential functions from two

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		204.d.i 204.d.iii		input-output pairs, even when the input pairs are not one unit apart
4.7	K	204.b.i 204.c.iv 204.d.i 204.d.iii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Radiocarbon dating **Math focus:** Using the half-life of elements to calculate how much of the element remains over time
4.8	K	204.b.i 204.c.iv 204.d.i 204.d.iii	LP1, LP2, LP3, LP6, LP7	**Context:** Bacteria successively splitting in a petri dish **Math focus:** Approximating the value of unknown exponents
4.9	K	204.a.i 204.a.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Understanding that a logarithm is a way to represent an exponent in an exponential equation
4.10	K	204.a.i 204.a.ii 204.d.ii	LP1, LP2, LP3	**Context:** Purely mathematical **Math focus:** Evaluating logarithms

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
4.11	K	204.a.i 204.a.ii 204.d.ii	LP1, LP2, LP3, LP7	**Context:** Purely mathematical **Math focus:** Calculating the value of a logarithmic expression using estimation or technology
4.12	K	204.c.i 204.d.i	LP1, LP2, LP3, LP5, LP7	**Context:** Moldy growth **Math focus:** Understanding the number e is an irrational constant
4.13	K	204.b.iii 204.c.i 204.d.i 204.d.iii	LP1, LP2, LP3, LP6, LP7	**Context:** Population of a colony of insects **Math focus:** Understanding that e is used in exponential models when the growth rate is applied at every moment
4.14	K	204.a.ii 204.d.ii	LP1, LP2, LP3, LP7	**Context:** Purely mathematical **Math focus:** Using base 10 and natural logarithms to solve exponential equations
4.15	M Amplify Activity 15.3 to introduce the graph of a logarithmic function and discuss	204.b.ii 204.c.i 204.c.ii 204.d.ii 204.d.iii	LP1, LP2, LP3, LP6, LP7	**Context:** Retire a millionaire; cicada population **Math focus:** Solving exponential equations using graphs or logs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	key features of the graph to attend to CPE 204.b.ii and 204.c.ii.			
4.16	K	204.b.iii 204.c.i 204.d.iii	LP1, LP2, LP3, LP6	**Context:** Two bank accounts; bacteria in different conditions; populations of two countries **Math focus:** Calculating and interpreting the intersection of the graphs of two exponential functions
4.17	K	204.a.i 204.a.ii 204.b.ii 204.b.iii 204.c.ii 204.c.iii	LP1, LP2, LP3, LP6, LP7	**Context:** Bacteria growth—doubling every hour; growing by a factor of 10 every day **Math focus:** Interpreting logarithmic functions in context
4.18	M Lesson listed as optional by the authors, but a good opportunity to address CPE connected to logarithmic functions as listed. Consider	204.b.ii 204.c.ii 204.c.iii	LP1, LP2, LP3, LP7	**Context:** How acidic is it; pH ratings; measuring earthquake strength **Math focus:** Understanding how logarithms are used to measure acidity and the intensity of earthquakes

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	asking questions not present in curricular notes to address the full scope of listed indicators.			
Modeling Task- Exponential Situations	M Consider using the situations offered as a menu of topics from which students can conduct research, gather data, and analyze for appropriate mathematical models to broaden the opportunity to engage in LPs listed.	204.b.i 204.b.ii 204.c.i 204.c.iii 204.c.iv	LP1, LP2, LP3, LP4, LP5, LP6, LP7	**Context:** Number of cars in a country; population changes; bacteria population; carbon dating; half-life; changes in quantity of books in a library **Math focus:** Asking and answering questions about a situation that can be modeled with an exponential function

M205: Modeling with Trigonometric Functions

Illustrative Mathematics–Geometry Unit 7: Circles and Algebra 2 Unit 6: Trigonometric Functions

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4).** This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one's thinking has excellent value in solidifying one's knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense

of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you…
205.a: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	i. use the definition of a radian to explain radian measure.
	ii. explain what an angle given with a measure in degrees would be in radians.
205.b: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers.	i. explain how radians lead to a dimensionless number that makes it possible to extend the domain of trigonometric functions.
	ii. use the unit circle to justify the value of sine, cosine, or tangent for an angle that is greater than 90 degrees.
205.c: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	i. describe the symmetry of a trigonometric function.
	ii. use the unit circle to explain why a trigonometric function has symmetry.
	iii. describe the periodicity of a trigonometric function.
	iv. use the unit circle to explain why a trigonometric function is periodic.
205.d: Analyze trigonometric functions using different representations.	i. create the graph of a trigonometric function.
	ii. identify key features of the graph of a trigonometric function, such as its period, amplitude, and frequency, and connect these features back to the context of the problem.
	iii. explain the relationship between the parameters of a trigonometric function and its graph.

Content and Practice Expectations	Indicators Choose an artifact where you…
205.e: Use trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	i. create a trigonometric function to model a real-world situation.
	ii. use a trigonometric function to answer a question about a real-world situation.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	MT
205.a.i	X				X																	
205.a.ii		X			X																	
205.b.i				X		X	X	X	X				X									
205.b.ii				X		X	X	X	X			X	X									
205.c.i						X	X	X	X				X									
205.c.ii			X				X	X	X				X									
205.c.iii			X							X		X	X	X				X	X	X		
205.c.iv			X									X	X	X								
205.d.i											X	X	X	X	X	X	X	X	X	X	X	X
205.d.ii											X	X	X	X	X	X	X	X	X	X	X	X
205.d.iii											X	X	X	X	X	X	X	X	X	X	X	X
205.e.i									X						X	X				X	X	X
205.e.ii									X						X	X		X		X	X	X

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
G.7.11 (Use Lesson 11 from Unit 7 of Geometry Course)	K	205.a.i	LP2, LP3, LP5	**Context:** Purely mathematical **Math focus:** Defining radian measure as the ratio of the arc length to the circle's radius
G.7.12	K	205.a.ii	LP2, LP3	**Context:** Pie coloring **Math focus:** Understanding angles measured in radians
A2.6.1 (Transition to Algebra 2 Unit 6 Lessons)	K The content lays a foundation for identified CPEs.	205.c.ii 205.c.iii 205.c.iv	LP2, LP3, LP6	**Context:** Ladybug on clock's second hand **Math focus:** Using Pythagorean theorem to find coordinates of points on a circle centered at origin; introduces the concept of periodic functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.2	K The content lays a foundation for identified CPEs.	205.b.i 205.b.ii	LP2, LP3	**Context:** Purely mathematical **Math focus:** Using trigonometry to express the coordinates of a point in quadrant 1, one unit away from origin
6.3	K	205.a.i 205.a.ii	LP1, LP2, LP3, LP6	**Context:** Bicycle wheel **Math focus:** Unit circle—radian angle measurements and defining points by coordinates or angle of rotation
6.4	M Consider asking students to describe the connection between the coordinates to trig functions within this lesson to address CPE 205.b.	205.b.i 205.b.ii 205.c.i	LP2, LP3	**Context:** Purely mathematical **Math focus:** Unit circle—finding angles and estimating coordinates
6.5	M Teaching note: the idea of symmetry is implied but not directly stated. Be sure to discuss symmetry of the trig	205.b.i 205.b.ii 205.c.i 205.c.ii	LP2, LP3	**Context:** Purely mathematical **Math focus:** Pythagorean identity—calculating values of coordinates given one; direct work with unit circle

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	function to address CPE 205.c.i.			
6.6	K	205.b.i 205.b.ii 205.c.i 205.c.ii	LP2, LP3, LP5	**Context:** Purely mathematical **Math focus:** Pythagorean identity—finding the values of cosine, sine, and tangent if one is known
6.7	M Components of this lesson build a foundation for CPE 205.e, since students work with trig expressions, not functions.	205.b.i 205.b.ii 205.c.i 205.c.ii 205.e.i 205.e.ii	LP1, LP2, LP3, LP6	**Context:** Clock/ferris wheel **Math focus:** Using cosine and sine to figure out information about points rotating in circles
6.8	K	205.c.iii	LP2, LP3, LP5	**Context:** Periodic functions in context (distance from startline, temperature, and population) **Math focus:** Understanding that the graph of a periodic function can look like a wave whose outputs repeat between the same max and min values

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.9	K	205.d.i 205.d.ii 205.d.iii	LP1, LP2, LP3, LP7	**Context:** N/A—use of applet to obtain coordinates, given an angle **Math focus:** Using coordinates of points on the unit circle to graph the cosine and sine functions
6.10	K	205.b.ii 205.c.iii 205.c.iv 205.d.i 205.d.ii 205.d.iii	LP2, LP3, LP6	**Context:** Windmill blade **Math focus:** Understanding how to find the values of cosine and sine for inputs greater than 2pi radians
6.11	K	205.b.i 205.b.ii 205.c.i 205.c.ii 205.c.iii 205.c.iv 205.d.i 205.d.ii 205.d.iii	LP1, LP2, LP3, LP7	**Context:** Windmill; clock hand **Math focus:** Understanding how to find the values of cosine and sine for inputs less than 0 radians

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.12	K	205.c.iii 205.c.iv 205.d.i 205.d.ii 205.d.iii	LP2, LP3	**Context:** Purely mathematical **Math focus:** Understanding why the tangent function has a period of pi; understanding why the graph of tangent has asymptotes; exploration of tangent graph
6.13	K	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP2, LP3, LP6, LP7	**Context:** Windmill blade **Math focus:** Writing trigonometric functions to represent situations with different amplitudes and midlines
6.14	M Teaching note: ask students to create equations in Activity 14.3 (Spinning Fan).	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP6	**Context:** Windmill blade; spinning fan **Math focus:** Graphing horizontal translations of trigonometric functions; using amplitude and midline to describe a situation
6.15	K	205.d.i 205.d.ii 205.d.iii	LP2, LP3, LP7	**Context:** Sound waves; matching activity **Math focus:** Identifying midline, amplitude, and horizontal translations given a graph or equation

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.16	K	205.c.iii 205.d.i 205.d.ii 205.d.iii 205.e.ii	LP1, LP2, LP3, LP6	**Context:** Ferris wheel **Math focus:** Finding the period using an equation or graph
6.17	K	205.c.iii 205.d.i 205.d.ii 205.d.iii	LP1, LP2, LP3, LP5	**Context:** Purely mathematical **Math focus:** Creating an equation of a trigonometric function using information about its graph
6.18	K	205.c.iii 205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP5, LP6	**Context:** Bike wheel; carousel **Math focus:** Representing a circular motion situation using a graph and an equation
6.19	K	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Visibility of the moon **Math focus:** Creating a model of data that is approximately periodic and using the model to make predictions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task- Swept Away	K Two versions are given—the second version has a few more questions analyzing the situation before asking students for a model. Consider using local data to increase degree of relevance.	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP5, LP6, LP7	**Context:** Tides in Boston **Math focus:** Modeling with trigonometric functions

Appendix A: Change Log

Date	Change
8/10/2025	In M102, added an indicator to CPE 102.d to clarify that when working with functions, the concepts of independent and dependent variables is important and further distinguish this CPE from 102.h.
8/10/2025	In M152, added CPE 152.d to reflect missing content for the badge and re-coded the remaining CPEs to match the new order.
8/10/2025	In M101 and M102, added a note about badging sites potentially combining these into a single badge if they are using off-the-shelf curricula that make it hard to disentangle the concepts and skills portion of linear functions and equations from the modeling portion.
8/1/2025	Added indicator M 201.c.iii to include expectations for translating between recursive and explicit forms